Name: ____

PHYS 1220-02 Exam 4

Each question counts as one point unless otherwise indicated. (Point values are meaningful only when adding scores within the same standard: each standard counts the same toward your course grade.) You may use an 8.5"×11" note sheet written on both sides and a calculator. Please write your answers in the boxes provided. Show your work outside the boxes. If you need to change the answer you wrote in a box, erase it and write your intended answer. This is easiest if you write in pencil. You have 110 minutes.

Some physical constants you may find useful are

Coulomb constant	k	$8.987 \times 10^{9}\,\frac{\mathrm{N}\cdot m^2}{\mathrm{C}^2}$
Vacuum permittivity	ε_0	$8.854 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$
Elementary charge	e	1.602×10^{-19} C
Vacuum permeability	μ_0	$4\pi \times 10^{-7}\,\frac{\mathrm{N}}{\mathrm{A}^2}$

1. (10 points) Match each of Maxwell's equations to its summarized meaning. Draw a line from each formula on the left to the matching description on the right.

Gauss's law for electric charge

$$\oint_A \vec{E} \cdot d\vec{A} = q_{\mathrm{encl}}/\varepsilon_0$$

Gauss's law for magnetic monopoles

$$\oint_A \vec{B} \cdot d\vec{A} = 0$$

Ampère's law

$$\oint_A \vec{B} \cdot d\vec{l} = \mu_0\left(I_{\mathrm{encl}} + \varepsilon_0 \frac{d}{dt}\int_A \vec{E} \cdot d\vec{A}\right)$$

Faraday's law

$$\oint_A \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A}$$

A changing magnetic field creates a circulating electric field.

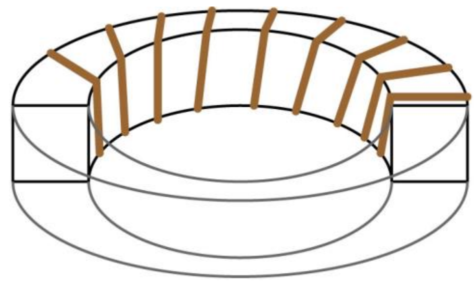
Electric currents and changing electric flux create circulating magnetic fields.

Magnetic monopoles do not exist.

An electric charge creates a field pointing away from itself if the charge is positive, or toward itself if the charge is negative.

2. In the previous exam, you found that the formula for the magnetic field B inside a toroid electromagnet was $B = \frac{\mu_0 NI}{2\pi r}$, where r is the distance from the principal axis of the toroid, N is the number of windings, and I is the current.

Suppose a toroid electromagnet with square windings has N loops. Its tunnel has width and height a, and the toroid has an inner radius b. (Therefore, its outer radius is $b + a$.)

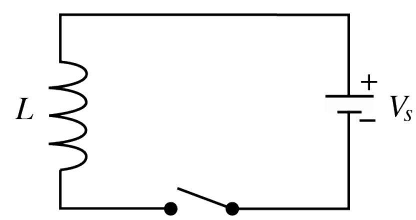


On Monday of Thanksgiving week, we derived a formula for the energy density u_B (energy per volume) of a magnetic field B.

A. (5 points) Tell me the formula for u_B in terms of B and any necessary constants.

B. (5 points) Let us use this u_B formula to find the energy U stored in this toroid when it carries a current I. We can find this energy by integrating the field energy density over the interior volume of the toroid. We can make a differential volume element an annulus of the toroid with an inner radius r, thickness dr, (outer radius $r + dr$), and width a, giving a volume $dV = 2\pi r a dr$. If we know the energy density of the magnetic field u_r at radius r, we can find the toroid's energy by $U = \int_b^{b+a} u_r dV$. Tell me this formula for the toroid's magnetic field energy in terms of the toroid's features (a, b, N, I) and any appropriate physical constants. If you don't have a formula for u_r, make one up and tell me what it is. If your formula for the toroid's energy is consistent with your formula for u_r, you will get credit for this section, even if your u_r is incorrect.

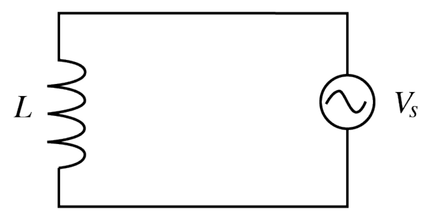
3. In the circuit at right, an ideal voltage source, an inductor, and a switch are connected in a series circuit. The voltage source produces a steady voltage of V_s. The switch closes at time $t = 0$ and remains closed.



   A. (3 points) Sketch a plot of the current I (vertical axis) through the circuit over time (horizontal axis).

   B. (2 points) Tell me the formula for I as a function of time in terms of the quantities V_s, L, and t.

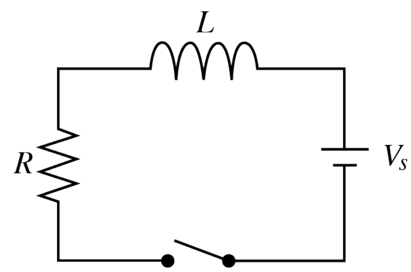
4. In the circuit at right, an alternating voltage source powers an inductor. The voltage source produces a time-varying voltage $V = V_s \cos(\omega t)$.



   A. (3 points) Sketch a plot of the current I (vertical axis) through the circuit over time (horizontal axis).

   B. (2 points) Tell me the formula for I as a function of time in terms of the quantities V_s, L, ω, and t.

5. In the circuit at right, an ideal voltage source, an inductor, a switch, and a resistor are connected in a series circuit. The voltage source produces a steady voltage of V_s. The switch closes at time $t = 0$ and remains closed.



A. (1 point) What is the current an instant after time $t = 0$? Express your answer in terms of V_s, R, L, and any appropriate constants.

B. (1 point) What is the current a very long time after the switch is closed, at $t = \infty$? Express in terms of V_s, R, L, and any appropriate constants.

C. (2 points) At what time is the current halfway between its values at $t = 0$ and at $t = \infty$? We'll call this time $t_{½}$. Express in terms of V_s, R, L, and any appropriate constants.

D. (3 points) What is V_L, the voltage across the inductor, at times $t = 0$, $t = t_{½}$, and $t = \infty$? Express in terms of V_s, R, L, and any appropriate constants.

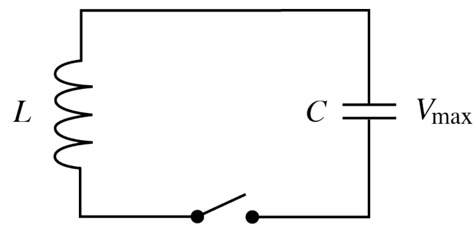
$t = 0$: $t = t_{½}$: $t = \infty$:

E. (3 points) What is V_R, the voltage across the resistor, at times $t = 0$, $t = t_{½}$, and $t = \infty$? Express in terms of V_s, R, L, and any appropriate constants.

$t = 0$: $t = t_{½}$: $t = \infty$:

6. A 100 μF capacitor is connected to a 360 μH inductor.

   A. (5 points) What is the resonant frequency of this combination?



   B. (5 points) Suppose that when the circuit is in resonance its maximum voltage is 5.00 volts. What is the maximum current during the resonant cycle?

7. A transformer steps down voltage of 2.40 kV at a power line to house service at 120 V.

   A. (5 points) If the house is drawing 20 amperes of current from the transformer, what is the current from the power line into the transformer?

   B. (5 points) If the transformer core has 600 windings in the secondary (house side) circuit, how many windings are in the primary (pole side) circuit?