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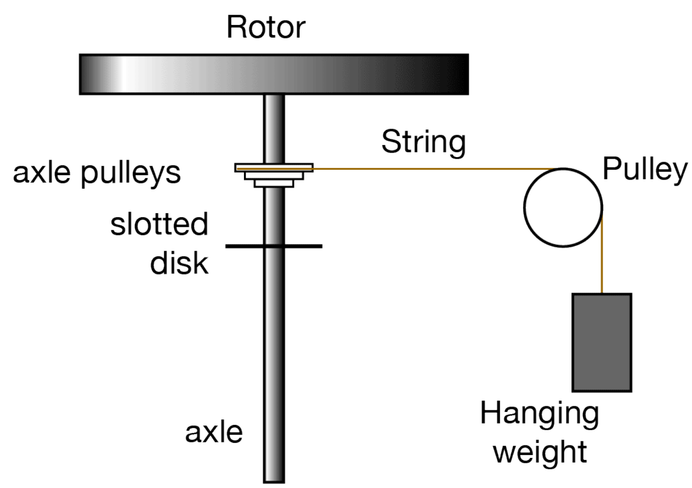
LAB 9. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you will use a falling mass to pull a string, which will generate a torque on a pulley to accelerate a heavy rotor with an unknown moment of inertia. You will measure the acceleration of the rotor to determine the rotor's moment of inertia.

Apparatus

The rotor is mounted in the bearing with its axis vertical. The axle of the rotor is shared with three pulleys of different radius and a disk with ten slots. One end of a string is wound around one of the pulleys, so that pulling the string horizontally away from the pulley applies a torque about the axis. The string is placed over an external pulley and the other end is attached to a hanging mass, so that the weight of the hanging mass pulls on the string. An auxiliary mass can be placed on the rotor to increase its inertia. A photogate can be installed around the slotted disk to measure its rotation.



Theory

As the hanging mass falls, it turns the axle of the rotor, so that the motion of the hanging mass and the rotor are linked. When the rotor and pulley of radius r turn through an angle θ radians, the string and attached weight move a vertical distance $d = \theta r$. Correspondingly, when the rotor turns at angular speed ω radians per second, the weight moves at speed $v = \omega r$; when the rotor's angular acceleration is α radians per second per second, the hanging weight accelerates with acceleration $a = \alpha r$.

The force pulling the mass downward is its gravitational attraction mg to the earth. Opposing its fall is the tension T in the string suspending it. The rotation of the rotor is driven by the torque $\tau = rT$ applied by the same string via the axle pulley of radius r. Inertia is provided by the mass m of the hanging weight and the rotational inertia I of the rotor. The net force on the hanging mass is a downward $\sum F = mg - T$. The acceleration of the hanging mass is then $a = \sum F/m = g - T/m$, and the angular acceleration of the rotor is $\alpha = \tau/I$. Combining these formulas gives, after some algebra, $\alpha = \frac{mgr}{mr^2+I}$.

The photogate detects if its light path is blocked or open. By counting the times the photogate's open/blocked state alternates, Logger Pro calculates the angular distance θ moved by the slotted disk. It detects only the absolute value of changes in θ; it has no information about the direction of $\Delta\theta$, or consequently of ω or α. You need to provide that information from your knowledge of the situation.

Experiment

In this activity three different hanging masses will pull on the string, which is wound around the three different pulleys on the axle. In each of the nine cases, you will measure the acceleration of the rotor twice.

Supplies

Rotor apparatus, string, photogate "smart pulley", interface and computer with Logger Pro software and a spreadsheet installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler.

Pre-lab

Construct the data table for your experiment. Get it approved by your instructor before you begin collecting data. It's a good idea to also plan the spreadsheet you'll need for the analysis. Your data table can be part of the spreadsheet.

Data Collection

Setup

1. Measure the mass and diameter of the rotor.
2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.
3. Install the photogate around the slotted wheel and recognize the photogate in Logger Pro.
4. Mount the rotor onto the bearing.
4. Download and open the experiment file "Rotary Photogate." (This may take some assistance.) It will calculate and plot θ and ω of the slotted disk from the photogate input.

Measurements

1. Wind the string around one of the pulleys, leaving enough free to hang the pulling weight.
2. Hold the rotor to keep it from turning. Run the string over the pulley and hang a mass at the end of the string.
3. Start data collection. Release the rotor to allow the rotor and hanging weight to move.
4. Just before the falling mass reaches the floor, stop the rotor. Stop data collection.
5. Fit the linear portion of the ω-t plot with a linear trend line. The slope of this trend line is the angular acceleration α of the rotor.
6. Record the hanging mass, the pulley radius, the fitted acceleration (trendline slope), and the uncertainty u (±) of the angular acceleration.
7. Repeat each run. If the two accelerations are not within 5% of each other, measure a third time.
8. Measure the angular accelerations of each of the three masses using each of the three pulleys on the rotor.

Extension

9. If time permits, repeat with the auxiliary ring mass added to the rotor.

Data Processing

1. Make a spreadsheet that collates your data in a table. Each row of the table should be a single run. In the columns, enter all the measured quantities for the runs: r, m, and α.
2. In another column in the spreadsheet, calculate the predicted acceleration α_c using the formula provided.
3. What? You need to know the rotor's moment of inertia I to calculate its acceleration? Interesting. What can you do about that? You *can* calculate I from each measurement, by solving the equations of motion for I. But then you have an I for each run, and they will inevitably be different from each other. But there is only one rotor, and it only has a single I about its axis. So how do you find the one overall best estimate of I?
4. Oh, I have an idea! The point of this lab is to figure out what the rotor's inertia is. Perhaps you can make a guess of I and calculate what all the accelerations would be with that I. You can test how well the predicted angular accelerations α_c match the measured accelerations. Then, you can try out different values of I to find the one that makes the predictions come out closest to the measurements.
5. Of course, doing that means that you need some way to quantify how close the predicted accelerations are to the measured ones. For a single acceleration, it can just be the "residual" difference $\alpha_{ci} - \alpha_i$. You can summarize that in a single number for the whole data set—the sum of squares of residuals $S = \sum_1^N (\alpha_{ci} - \alpha_i)^2$. Set up the spreadsheet to calculate that "goodness of fit" score S.
6. Keep records of the I values you have tried and their corresponding goodness-of-fit scores. You can't know that you have the optimal value without other numbers to compare it to.
7. An optimal I parameter giving a best overall fit is useful, but it doesn't tell you if the model actually fits the data well. Find a way to scale the fit score to give a rough idea of how close an average calculated acceleration is to the corresponding measured acceleration.
8. Make a scatter plot of S vs. I, showing how well your different guesses of I predict the observed angular accelerations.
9. Using the best-fit value of I, make a scatter plot of the residuals $\alpha_{ci} - \alpha_i$ vs. mr.

Check-out

Show the instructor the data, the plot of S vs. I, and the residuals plot.