PHYS 1110 Discussion: Engines and entropy

Summary

Internal energy and the first law of thermodynamics

The **internal energy** U of a substance is the total kinetic energy of its molecules relative to its enter of mass plus their intermolecular potential energy. The **first law of thermodynamics** states that the change in internal energy of any system is given by $\Delta U = Q - W$, where

> Q is the internal energy added to the system by **heat** and
>
> W is the internal energy lost from the system by doing **work**.

Another equally valid convention is $\Delta U = Q + W$, where W is the work done *on* the system.

For an **isobaric** (constant P) process, $W = p\Delta V$. For an **isothermal** (constant T) process, $W = -nRT \ln(V_i/V_f)$. For an **isochoric** (constant V) process, $W = 0$.

Entropy

The **entropy** of a system is a measure of how dispersed its matter and energy are. The entropy *change* between states is related to the heat input in a *reversible* process linking the states: $\Delta S = Q_{\text{rev}}/T$.

Second law of thermodynamics: $\Delta S \geq 0$ for all processes that actually occur.

Entropy and temperature

The **temperature** T of an object is its tendency to lose energy by transferring heat. It is related to the object's tendency to gain entropy by absorbing heat: $1/T = \lim\limits_{\Delta U \to 0} \Delta S/\Delta U$.

The entropy change of an infinitesimal process then is $\Delta S = \Delta U/T = q/T$.

Efficiency of an engine

The **efficiency** of an engine is the work produced divided by the heat input from the hot reservoir. If the engine draws heat Q_h at high temperature T_h and discards heat Q_c at a lower temperature T_c, the work it does is $W = Q_h - Q_c$. Its efficiency is

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

W as defined here is the total work done *by* the engine in its entire cycle while Q_h is the heat *gained* by the engine form the hot reservoir and Q_c the heat *expelled* by the engine to the cold reservoir. The sign conventions are different from those used in the first law!

For total entropy to increase, the entropy gained by the cold reservoir Q_c/T_c is at least as great as the entropy lost by the hot reservoir Q_h/T_h. This limits the engine's efficiency to $e \leq 1 - T_c/T_h$.

Performance of a refrigerator or heat pump

The **coefficient of performance** COP of a refrigerator is the heat extracted from the cold body divided by the work input, COP = Q_c/W. (Signs are opposite what they are for engines.) For a heat pump, COP = Q_h/W. The second law limits the performance to at best

$$\text{COP} \leq \frac{T_c}{T_h - T_c} \text{ for a refrigerator, and COP} \leq \frac{T_h}{T_h - T_c} \text{ for a heat pump.}$$

Problems

1. A system receives 100 J of heat and does 125 J of work on the surroundings.
   a. Is this possible?
   b. Give a reason for your answer.
2. For each of the following irreversible processes, explain how you can tell that the total entropy of the universe has increased. Has matter spread out? Has energy spread out?
   a. Stirring salt into a pot of soup.
   b. Scrambling an egg.
   c. A wave hitting a sand castle.
   d. Cutting down a tree.
   e. Burning gasoline in an automobile.
3. An ice cube (mass 30 g) sits on the kitchen table, where it gradually melts. The temperature in the kitchen is 25 °C.
   a. Calculate the heat transferred from the kitchen to the ice at 0 °C. The latent heat of fusion of water is 3.33×10^5 J/kg.
   b. Calculate the change in the entropy of the ice cube as it melts into water at 0 °C. (Don't worry about the fact that its volume changes a little.)
   c. Calculate the change in the entropy of the kitchen as the ice cube melts.
   d. Calculate the total entropy change of the universe to melt the ice.
   e. Is melting the ice spontaneous?
4. In places where it gets hot in the summer (unlike Laramie), it is common to install air conditioners to cool living spaces.
   a. Why must you put an air conditioner in the window of a building, rather than in the middle of a room?
   b. Can you cool off your kitchen by leaving the refrigerator door open? Explain.
   c. Estimate the maximum possible coefficient of performance of a household air conditioner. Use any reasonable values for the reservoir temperatures.
5. At a power plant that produces 1 GW (10^9 watts) of electricity, the steam turbines take in steam at a temperature of 500 °C, and the waste heat is expelled into the atmosphere at 20 °C.
   a. What is the maximum possible efficiency of this plant?
   b. How many kilowatt hours of electrical energy does this plant produce in a year?

Suppose you install pipes and turbines made from a new material that allows the maximum steam temperatures to be raised to 600 °C.

c. What is the maximum possible efficiency of a plant that takes in steam at 600 °C and expels waste heat to the atmosphere at 20 °C?

d. How many kilowatt hours of electrical energy will the plant produce in a year from the same fuel consumption (heat input Q_h) as before?

e. How much money will you make in a year by selling the additional electricity for 5 cents per kilowatt hour?