Discussion 11: Waves

Summary

Waves

A wave is an organized disturbance that propagates in time and space. In a **transverse** wave, the medium oscillates in a direction that is perpendicular to the propagation direction of the wave; in a **longitudinal** wave, the medium oscillates in the same direction that the wave propagates.

The simplest mathematical formula to describe wave motion is a sinusoidal function

$$y = A\cos(kx - \omega t + \varphi)$$

where A is the **amplitude** of the wave, $k = 2\pi/\lambda$ is the **wave number**, $\omega = 2\pi/T$ is the **angular frequency**, and φ is the **phase offset**. Here, T is the **period**, or repeat time of the wave; and λ is the **wavelength**, or repeat distance of the wave. The **propagation speed** of the wave is given by $u = \lambda/T = \omega/k$.

Transverse waves in a cord

The propagation speed of a wave in a cord of length density (mass per unit length) μ under tension F is $u = \sqrt{F/\mu}$.

Sound Intensity

Intensity

Sound from a point source produced with power P has an intensity at distance r of $I = \frac{P}{4\pi r^2}$.

Decibel scale

The decibel scale is a logarithmic transformation of sound intensity. A sound with an intensity I has a decibel rating of $\beta = (10 \text{dB}) \log_{10}(I/I_0)$, where I_0 is the reference intensity of 10^{-12} W/m^2 .

The difference in decibel ratings of two sounds with intensity I_1 and I_2 is

$$\beta_2 - \beta_1 = (10 \text{dB}) \log_{10}(I_2/I_1)$$

The difference in decibel rating of sound from the same source observed at two different distances r_1 and r_2 is

$$\beta_2 - \beta_1 = (20 \text{dB}) \log_{10}(r_1/r_1)$$

Doppler shift

The frequency f_D of sound received by the detector depends on the source frequency f_S , the velocity of the sound v, and the velocity of the source v_S and detector v_D . Adopting the convention that the positive direction is from the source toward the detector, the detected frequency is

$$f_D = f_S \frac{v - v_D}{v - v_S}$$

Problems

- 1. The lowest note on a grand piano has a frequency of 27.5 Hz. The string is 2 m (more or less; let's just assume it is exactly 2.000 m) long, and its tension is 1000 N (again, more or less, so let's say it's exactly 1000 N).
 - a. What is the wavelength of the wave in the string?
 - b. What is the propagation speed of the wave in the string?
 - c. What must be the length density μ (mass per unit length, in units of kg/m or g/m) of the string?
 - d. What must the mass of the string be?
 - e. If the string is to be made of steel, which has a volume density $\rho = 7800 \text{ kg/m}^3$, what must the volume of the string be?
 - f. The volume of a cylinder of length L and diameter d is $V = \pi d^2 L/4$. What must the diameter of the string be?
 - g. Is this "string" likely to flex like a string? Why or why not?
 - h. How are the low strings on a piano actually constructed to give them the proper length densities?
- 2. The sound level at a Formula 1 race is reported to reach 140 decibels at trackside, about 5 m from the cars. Long-term exposure to sound levels above 85 dB damages hearing; sound levels above 110 dB are uncomfortable, and levels above 130 dB are painful.

Reggie Racefan has arrived at a Formula 1 race without his earplugs, and he doesn't want to pay for the expensive earplugs for sale at the race.

- a. How far from the track must Reggie stand to keep the noise he hears below the discomfort level of 110 dB?
- b. How far from the track must be stand to keep the noise he hears below 85 dB?
- 3. A spectator at a Formula 1 race (with proper hearing protection) notices that the sound from an approaching car has a frequency of 380 Hz, while the sound from the same car driving away has a frequency of 240 Hz. (The sound from a car is a combination of many frequencies and phases, but 380 and 240 Hz are the intensity maxima.) The speed of sound in air is 342 m/s
 - a. What is the speed of the car? Assume that it has the same speed when it is driving toward the spectator as when it is driving away.
 - b. What is the frequency of the sound emitted by the car?