PHYS 1110 Discussion: Rigid-body Rotation

Summary

Rigid bodies

Rigid bodies do not change shape, size, or distribution of mass during a process. Their motion can always be described as a combination of translation of the center of mass and rotation about the center of mass.

Rotational kinematics

Radians

The extent of rotation about an axis is most naturally described in terms of **radians**, a dimensionless unit defined as displacement of a point through an arc length equal to its distance from the axis of rotation. The measure of any angle in radians is $\theta = s/r$, where s is the arc length and r is the distance from the axis.

Velocity and acceleration

Angular velocity $\omega = \Delta\theta/\Delta t = v/r$, where $v = \Delta s/\Delta t$

Angular acceleration $\alpha = \Delta\omega/\Delta t = a_{\parallel}/r$, where $a_{\parallel}$ is the component of acceleration tangent to the path

Constant-α motion

$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$

Problems

1. One design of bicycle before drive chains were adopted was the *pennyfarthing*: a bicycle with a large front wheel whose axle was also the pedal crank, and a much smaller rear wheel. A typical pennyfarthing might have a front wheel with a radius of 0.72 meters and a rear wheel with a radius of 0.12 meters.

   Suppose that a pennyfarthing bicycle with these dimensions travels 100.0 meters.

   A. How many revolutions does its front wheel rotate when the bicycle travels 100 meters?

   B. How many radians does its front wheel rotate when the bicycle travels 100 meters?

   C. How many revolutions does its rear wheel rotate when the bicycle travels 100 meters?

   D. How many radians does its rear wheel rotate when the bicycle travels 100 meters?

   Suppose the pennyfarthing starts down a hill at a speed of 2.0 meters per second and accelerates downhill at a rate of 0.16 meters per second per second.

   E. What is the initial angular speed of its front wheel, in radians per second?

   F. What is the initial angular speed of its rear wheel, in radians per second?

G. What is the angular acceleration of its front wheel, in radians per second per second?

H. What is the angular acceleration of its rear wheel, in radians per second per second?

I. What is the bicycle's speed, in meters per second, after traveling 100 meters?

J. What is the angular speed of its front wheel after traveling 100 meters?

K. What is the angular speed of its rear wheel after traveling 100 meters?

2. (Your textbook, problem 7-61.) In a home laundry dryer, a cylindrical drum containing wet clothes is rotated steadily about a horizontal axis. So that the clothes will dry uniformly, they are made to tumble. The rate of rotation of the drum is chosen so that a garment will lose contact with the drum when the garment is at an angle of 68.0° above the horizontal. If the radius of the drum is 0.330 m, what must its rate of rotation be?