Worksheet 2: Trajectory Kinematics

Objectives

- Represent position, velocity and acceleration as vectors.
- Work with ballistic trajectories in the vertical and horizontal directions.

Summary

Position, velocity, and acceleration vectors

Position $\vec{r} = (x, y, z)$

Velocity $\vec{v} = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta x}{\Delta t}, \dfrac{\Delta y}{\Delta t}, \dfrac{\Delta z}{\Delta t} \right) = (v_x, v_y, v_z)$

Speed $v = \|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Acceleration $\vec{a} = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta v_x}{\Delta t}, \dfrac{\Delta v_y}{\Delta t}, \dfrac{\Delta v_z}{\Delta t} \right) = (a_x, a_y, a_z)$

Component of $\vec{a}$ parallel to $\vec{v}$: $a_\| = $ rate of change of speed $= \Delta v / \Delta t$

Component of $\vec{a}$ perpendicular to $\vec{v}$: $a_\perp$. Affects only direction of $\vec{v}$.

Projectiles

When the only force is gravity (no air resistance, etc.), the horizontal (x) and vertical (y) components of the motion can be treated independently. For a projectile launched from (x_0, y_0) with initial speed v_0 at angle θ above horizontal, the initial velocity $\vec{v}_0 = (v_{0x}, v_{0y}) = (v_0 \cos\theta, v_0 \sin\theta)$, and

$$a_x = 0 \qquad\qquad a_y = -g$$

$$v_x = v_{0x} \qquad\qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t \qquad\qquad y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

These formulas require that the $+y$ direction be up. The magnitude of the acceleration due to gravity at Earth's surface is approximately 9.8 m/s^2, designated as g. Similar formulas apply for different coordinate systems.

Range Equation

The horizontal distance traveled by a projectile before landing at its launch height y_0 is $v_0{}^2 \sin 2\theta / g$.

Problems

There is not room on this worksheet for your work. Use scratch paper.

1. A projectile is launched from height h at speed v_0 and angle θ above the horizontal and lands on the ground.

   a. Draw a diagram of the initial situation. Show axis directions and the location of the origin.

   b. Find the formula for the maximum height reached by the projectile.

   c. Find the formula for the time at which the projectile reaches the top of its arc.

   d. Find the formula for the projectile's velocity $\vec{v} = (v_x, v_y)$ at the top of its arc.

   e. Find the formula for the vertical component of the projectile's velocity just as it reaches the ground.

   f. Find the formula for the time at which the projectile lands on the ground.

   g. Find the formula for the horizontal distance the projectile travels to the top of its arc.

   h. Find the formula for the horizontal distance the projectile travels to land on the ground.

   i. The projectile is a car tire fired from a hobbyist's catapult at a speed of 20.0 m/s at an angle of 50° above horizontal from a height of 2.50 m above the ground. How far away horizontally from the launch does it land on the ground?

2. A physics student throws a rubber duck from an apartment balcony that is 30.0 meters above the ground. The duck strikes the ground 1.94 seconds later at a distance of 15.5 meters from the wall of the apartment building.

   a. What was the initial velocity of the rubber duck?

   b. If you have time, you can also find the velocity of the rubber duck just as it strikes the ground.

   Even though the question doesn't tell you to find an algebraic solution first, I want you to find an algebraic solution first.