PHYS 1110 Exam 4

Brief Solutions

1. Exoplanetary orbit

We are given the mass $M = 5.00 \times 10^{29}$ kg of the star and the period $T = 2.00 \times 10^6$ s of the planet's orbit, and asked to find the orbital distance r. If the orbit is uniform circular motion and the force acting on the planet is the star's gravity, we have

$$m\frac{4\pi^2 r}{T^2} = G\frac{Mm}{r^2}$$
$$4\pi^2 r^3 = GMT^2$$
$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{(6.674 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}})(5.00 \times 10^{29}\,\mathrm{kg})(2.00 \times 10^6\,\mathrm{s})^2}{4\pi^2} = 3.38 \times 10^{30}\,\mathrm{m}^3$$
$$r = 1.50 \times 10^{10}\,\mathrm{m}$$

2. Orbital formulas

A. Kinetic energy formula

Again, starting with uniform circular motion under the force of gravity,

$$m\frac{v^2}{r} = G\frac{Mm}{r^2}$$
$$K = \frac{1}{2}mv^2 = G\frac{Mm}{2r}$$

B. Gravitational potential energy

Here, we just need to use the formula for gravitational potential energy.

$$U_g = -G\frac{Mm}{r}$$

C. Escape energy

The planet would need enough kinetic energy for its total mechanical energy to exceed zero.

$$0 \leq E - G\frac{Mm}{r}$$
$$E = G\frac{Mm}{r}$$

D. K/U_g ratio

$$K/U_g = \frac{GMm/2r}{-GMm/r} = -1/2$$

E. K/E ratio

$$K/E = \frac{GMm/2r}{GMm/r} = 1/2$$

F. U_g/E ratio

$$U_g/E = \frac{-GMm/r}{GMm/r} = -1$$

3. Heat pack material

A. Specific heat capacity

To find the specific heat, we need to know the sample's mass, heat input, and temperature change.

$$Q = mc\Delta T$$
$$c = \frac{Q}{m\Delta T} = \frac{45.3\,\mathrm{kJ}}{(0.700\,\mathrm{kg})(25°\mathrm{C})} = 2.59\frac{\mathrm{kJ}}{\mathrm{kg}\,°\mathrm{C}}$$

B. Latent heat of melting

The latent heat of a phase change is just the energy absorbed or released per amount of material changing phase.

$$L = \frac{Q}{m} = \frac{73.5\,\mathrm{kJ}}{0.700\,\mathrm{kg}} = 105\,\mathrm{kJ/kg}$$

4. Sea level rise

We are asked about a linear expansion, but given information to calculate a volume expansion. To convert between the two, we need a relation between sea height and volume.

The volume of water in the top layer of the ocean is the layer's thickness h times its area A:$V = hA$. As the water expands, its height changes, but its volume remains constant: $\Delta V = (\Delta h)A$.

The formula for volume expansion is

$$\frac{\Delta V}{V_0} = \beta\Delta T.$$

We need to do a little algebra and substitute our known values of h_0, Δh, and β.

$$\Delta T = \frac{\Delta hA}{\beta h_0 A} = \frac{\Delta h}{\beta h_0} = \frac{0.0500\,\mathrm{m}}{(1.51\times 10^{-4}/°\mathrm{C})(500\,\mathrm{m})} = \frac{10^4}{1.51/°\mathrm{C}}\,\frac{5.00\times 10^{-2}}{5.00\times 10^2} = \frac{10^4\cdot 10^{-4}}{1.51}°\mathrm{C} = 0.66°\mathrm{C}$$

5. Expanding balloon

A. Work done by the balloon

The balloon is changing volume at constant pressure. The work it does is

$$p\Delta V = (95.0\times 10^3\,\mathrm{Pa})(1.814 - 1.700)\times 10^{-3}\,\mathrm{m}^3 = 10.83\,\mathrm{J}.$$

The unit transformation there might not be obvious; it comes from $\mathrm{Pa} = \mathrm{N/m^2} = \mathrm{J/m^3}$.

B. Change in internal energy

Here we use the first law of thermodynamics: $\Delta U = Q - W$. The heat absorbed by the balloon is $Q = 27.1\,\mathrm{J}$ and the work done *by* the balloon is 10.83 J. Thus the internal energy change is $\Delta U = (27.1 - 10.83)\,\mathrm{J} = 16.27\,\mathrm{J}$.

6. Free expansion

The answer is choice d. Compared to being in a larger volume, there are practically no ways for the molecules to all be in a smaller volume.

7. Heat transfer

The answer is choice d.

8. Heat engine

A. Efficiency

We are given $Qh = 1672\,\text{J}$, $Th = 525°\text{C} = 798.15\,\text{K}$, and $W = 800\,\text{J}$. That is enough to calculate the engine's efficiency, $e = W/Qh = (800\,\text{J})/(1672\,\text{J}) = 0.478$.

B. Waste heat

From $Qh = Qc + W$, so $Qc = Qh - W = (1672 - 800)\,\text{J} = 872\,\text{J}$.

9. Heat pump

We are given that $Tc = 25°\text{C} = 298.15\,\text{K}$ and $Th = 525°\text{C} = 798.15\,\text{K}$.

A. Coefficient of performance

The coefficient of performance of a heat pump is Qh/W. Its thermodynamic limit comes from the requirement that the total entropy change is positive.

$$\begin{gathered}\frac{Qh}{Th} \geq \frac{Qc}{Tc} \\ Qh\,Tc \geq (Qh - W)Th \\ Qh\,Tc \geq Qh\,Th - W\,Th \\ W\,Th \geq Qh(Th - Tc) \\ \frac{Th}{Th - Tc} \geq \frac{Qh}{W} \\ \text{COP} \leq \frac{Th}{Th - Tc}\end{gathered}$$

So the greatest possible COP is $Th/(Th - Tc) = 798.15/500 = 1.5963$. For a heat pump, this isn't very good. That is because the high temperature is much greater than the low temperature; heat pumps and refrigerators work best when the two temperatures are close.

B. Work to push 1000 J to the high-T reservoir

The COP is Qh/W, so $W = Qh/\text{COP} = (1000\,\text{J})/1.5963 = 626\,\text{J}$ and some change.