Lab 10. REACTANCE AND RESONANT CIRCUITS

1. Guiding Questions

- How are are an inductor's or capacitor's current and voltage related in an ac circuit?
- How is an inductor's or capacitor's behavior affected by ac frequency?
- How do LC and RLC circuits resonate?

2. Equipment

Inductor, capacitor, resistor box, variable-frequency ac source (signal generator), electric multimeter, breadboard, connecting wires

3. Background

1. Inductive Reactance

The properties of an inductor are complementary to those of a capacitor: the current through an inductor lags the voltage by 90°, and the inductive reactance is directly proportional to the ac frequency, $X_L = \omega L$.

Inductors are made from many turns of wire, so their resistance is not necessarily negligible. Thus an inductor's impedance Z_L is not pure reactance X_L, but contains components of both reactance and resistance R_L, so $Z_L^2 = X_L^2 + R_L^2$. The impedance Z_L and resistance R_L are fairly easy to determine experimentally, and the inductance can be calculated from those values.

Generalizing Ohm's law, we can express the peak or rms voltage of the inductor to its current (or vice versa) as

$$V_L = I_L Z_L.$$

Since there are no branches in a series circuit, all components pass the same current. In a series circuit containing an inductor and a resistor,

$$I = I_R = I_L$$

$$I_{rms} = \frac{V_{Rrms}}{R} = \frac{V_{Lrms}}{Z_L}$$

$$Z_L = R\,\frac{V_{Lrms}}{V_{Rrms}}$$

Then

$$X_L = \sqrt{Z_L^2 - R_L^2}$$

If we know ω, then we can determine the inductance $L = X_L/\omega$.

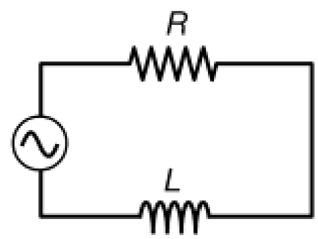
2. Resonant circuits

Circuits containing capacitors and inductors in series exhibit resonance at or near the frequency at which $X_L - X_C = 0$, which is $\omega_0 = 1/\sqrt{LC}$. Increasing resistance R in the circuit damps the resonance, with critical damping occurring when $2R^2 = L/C$.

4. Activities

In this lab, you will determine the reactance of an inductor and capacitor and measure the properties of resonant circuits. Your measurements of voltage will be on *alternating* current, so your multimeter should be set to AC voltage. The readings will be rms voltages.

1. Inductor



First, use the ohmmeter to measure the resistance R_L of the inductor as if it were a resistor and record it below. Place the inductor in series with a resistor and the signal generator. Investigate the circuit using sine waves at the indicated source frequencies. At each frequency, find a resistor (resistance R) that gives $V_{Rrms} \approx V_{Lrms}$. Record the measured values of R, V_{rms}, V_{Rrms}, and V_{Lrms}.

Be sure to include the units with your measurements.

DC resistance of inductor R_L: ______________________________

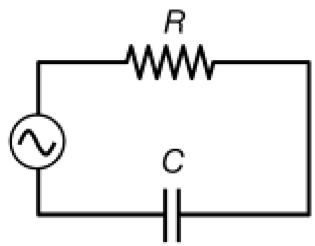
f (Hz)	R	V_{rms}	V_{Rrms}	V_{Lrms}	Z_L	ω	X_L	L
30								
100								
300								
1000								
3000								
10000								
30000								

Use the appropriate formulas (from above) to calculate the remaining columns. Estimate the inductance of the inductor.

L = ______________________________

2. Capacitor

You need a capacitor to go with your inductor to make a resonant circuit. Select a capacitor that should give you a fairly large capacitance, or connect several capacitors in parallel to give you a high capacitance. Place the capacitor in series with a resistor and the signal generator. Investigate the circuit using sine waves at the indicated source frequencies. At each frequency, find a resistor (resistance R) that gives $V_{Rrms} \approx V_{Crms}$. Record the measured values of R, V_{rms}, V_{Rrms}, and V_{Crms}.



Use only dielectric capacitors. Do not use an electrolytic capacitor, as they are ruined by currents of the wrong polarity.

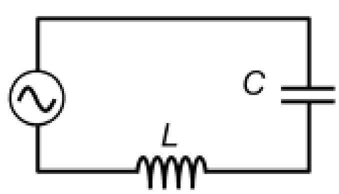
Description of capacitor: ____

f (Hz)	R	V_{rms}	V_{Rrms}	V_{Crms}	X_C	ω	C
30							
100							
300							
1000							
3000							
10000							
30000							

Use the appropriate formulas to calculate the remaining columns. Estimate the capacitance of the capacitor (or set of capacitors in parallel).

C: ____________________________________

3. LC Circuit



Connect the inductor and capacitor in series with the signal generator. Place the AC voltmeter in parallel with the capacitor, so that you measure the rms voltage V_{Lrms} across it. Scan the frequency of the signal generator to find the frequency giving the maximum voltage. This is the resonant frequency of the circuit. You may need to decrease the sensitivity of the voltmeter as you get close to the resonant frequency.

Record the value of this resonant frequency and also the voltage across the inductor and capacitor at this frequency. Repeat the measurements for other frequencies in the same order of magnitude. (For instance, if the resonant frequency is 500 Hz, you might want to measure the values also at 100 Hz, 250 Hz, 400Hz, 450 Hz, 550 Hz, 660 Hz, 1000 Hz, and 2500 Hz.) Calculate I_{rms} as $\omega C V_{Crms}$.

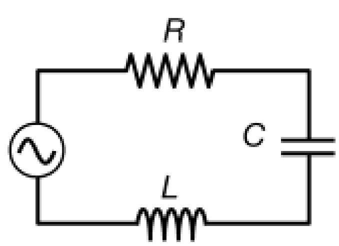
f (Hz)	V_{rms}	V_{Lrms}	V_{Crms}	ω	ω/ω_0	I_{rms}

Angular frequency of resonance ______________________ rad/s

Theoretical resonant frequency $\omega_0 = 1/\sqrt{LC}$ = ______________________ rad/s

Characteristic resistance $R_0 = \sqrt{L/C}$ = _______________________ Ω

4. RLC Circuits



Now add some resistance to the circuit (in addition to the intrinsic resistance of the inductor) by placing a resistor in series with the inductor and capacitor. Again, find the resonant frequency of the circuit by adjusting the source frequency to give the greatest V_C or V_L. Use three different resistances, one near the characteristic resistance $R_0 = \sqrt{L/C}$ and the other two below it. Fill in the remaining columns of the tables. Calculate I_{rms} as V_{Rrms}/R.

Resistance R_1: ________________________

f (Hz)	V_{rms}	V_{Lrms}	V_{Crms}	V_{Rrms}	ω	ω/ω_0	I_{rms}

Resistance R_2: ________________________

f (Hz)	V_{rms}	V_{Lrms}	V_{Crms}	V_{Rrms}	ω	ω/ω_0	I_{rms}

Resistance R_3: ________________________

f	V_{rms}	V_{Lrms}	V_{Crms}	V_{Rrms}	ω	ω/ω_0	I_{rms}

5. Lab Report

This doesn't have to be long, and it doesn't have to be fancy. What I want to see are:

- The data tables with measured and calculated quantities (this can be these original pages)
- A plot of inductive reactance X_L(vertical axis) vs. $\log_{10}(f)$ (horizontal axis)
- A plot of capacitive reactance X_C (vertical axis) vs. $\log_{10}(f)$ (horizontal axis)
- A plot of V_{Crms} and V_{Lrms} (vertical axis) vs. f (horizontal axis) for the LC circuit
- A single graph of I_{rms} vs. ω/ω_0 with a separate plot line for each of the different resistances used in the LC and RLC circuits. Label each curve with the resistance.

Write it up neatly and legibly.