Name: ______________________________

LAB 11. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you will use a falling mass to pull a string, which will generate a torque on a pulley to accelerate a heavy rotor with an unknown moment of inertia. You will measure the acceleration of the falling mass to determine the rotor's moment of inertia.

Apparatus

The rotor is mounted in the bearing with its axis vertical. The axle of the rotor is equipped with three pulleys of different radius. One end of a string is wound around one of the pulleys, so that drawing the string horizontally away from the pulley produces a torque about the axis. The string is placed over an external pulley and the other end is attached to a hanging mass, so that the weight of the hanging mass pulls on the string. An auxiliary mass can be attached to the rotor.

The pulley supporting the hanging mass is equipped with a photogate sensor, allowing the Capstone software to measure the speed of the string running over it.

Theory

As the hanging mass falls, it turns the pulley attached to the axle of the rotor, so that the motion of the hanging mass and the rotor are related. When the hanging mass falls a distance d, the pulley, of radius r, advances an angle $\theta = d/r$ radians. Correspondingly, when the mass falls at speed v, the pulley rotates at angular speed $\omega = v/r$; when the acceleration of the falling mass is a, the angular acceleration of the pulley is $\alpha = a/r$.

The only force promoting the descent of the mass is its gravitational attraction mg to the earth. Opposing the acceleration is its mass and the rotational inertia I of the rotor. The tension T in the string linking the rotor and the hanging mass determines their acceleration: the net force on the hanging mass is a downward $\Sigma F = mg - T$, and the only torque on the rotor is $\Sigma\tau = rT$. The acceleration of the hanging mass is then $a = \Sigma F/m$, and the angular acceleration of the rotor is $\alpha = \Sigma\tau/I$.

Experiment

In this activity you will pull on the string with three different hanging masses, and wind the string around the three different pulleys on the rotor axle. In each of the nine cases, you will measure the acceleration of the hanging mass.

Supplies

Rotor apparatus, string, photogate "smart pulley", PASCO interface and computer with Capstone software installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler.

Data Collection

Setup

1. Measure the mass and diameter of the rotor.
2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.

3. Connect the photogate pulley to the PASCO interface and recognize the smart pulley in Capstone.

Measurements

1. Wind the string around one of the pulleys, leaving enough free to hang the pulling weight.
2. Install the rotor into the bearing. Run the string over the photogate pulley and hang a mass at the end of the string.
3. Start data collection.
4. Just before the falling mass reaches the floor, stop the rotor. Turn off data collection.
5. Make a velocity-time plot of the data in Capstone. Fit the linear portion of the plot with a linear trend line. The slope of this trend line is the acceleration of the falling mass.
6. Record the hanging mass, the pulley radius, the fitted acceleration, and the uncertainty u (±) of the acceleration.
7. Repeat each run. If the two accelerations are not within 5% of each other, measure a third time.
8. Measure the acceleration of each of the three masses using each of the three pulleys on the rotor.

Extension

9. If time permits, repeat with the auxiliary ring mass added to the rotor.

Work-Up

Theory

Solve the equations above to find a formula for acceleration in terms of m, r, and I.

Data Processing

1. Make a spreadsheet that collates your data in a table. Each row of the table should be a single run. In the columns, enter all the measured quantities for the runs: r, m, and a.
2. In another column in the spreadsheet, calculate the predicted acceleration a_c using the formula you derived.
3. What? You need to know the rotor's moment of inertia I to calculate its acceleration? Interesting. What can you do about that? You *can* calculate I from each measurement, by solving the equations of motion for I. But then you have an I for each run, and they will inevitably be different from each other. But there is only one rotor, and it only has a single I about its axis. So how do you find the one overall best estimate of I?
4. Oh, I have an idea! The point of this lab is to figure out what the rotor's inertia is. Perhaps you can make a guess of I and calculate what all the accelerations would be with that I. You can test how well the predicted accelerations a_c match the measured accelerations. Then, you can try out different values of I to find the one that makes the predictions come out closest to the measurements.
5. Of course, doing that means that you need some way to quantify how close the predicted accelerations are to the measured ones. For a single acceleration, it can just be the difference

$a_{ci} - a_i$. Is there a way you can summarize that in a single number for the whole data set? When you figure it out, set up the spreadsheet to calculate that "goodness of fit" score.

6. You will need to keep track of the *I* values you have tried, and their corresponding goodness-of-fit scores. You can't know that you have the optimal value without other numbers to compare it to.
7. An optimal *I* parameter giving a best overall fit is useful, but it doesn't tell you if the model actually fits the data well. Find a way to scale the fit score to give a rough idea of how close an average calculated acceleration is to the corresponding measured acceleration.

Lab Report guidance

For this lab, make a standard lab report with the customary sections. Below are some considerations specific to this particular lab.

Abstract

What is the system? What quantities are you measuring directly? What quantities are you inferring from your measurements?

Purpose

This activity involves finding an unknown quantity without directly calculating it from a formula. What skills does this promote?

Theory

You need to derive a model for the acceleration based on the characteristics of the system. You also need to provide the theoretical basis for evaluating the goodness of fit of possible estimates of the rotor's moment of inertia. Explain all of this.

Experimental

What was your experimental apparatus? How did you take measurements? What steps did you take to minimize measurement errors?

Observation and Data

Your primary data should be in your notebook, but you also need to transcribe them to your spreadsheet.

Analysis and Discussion

How did you assign your optimal estimate of *I*? How did you evaluate the goodness of fit? What factors might affect the behavior of the system that are not accounted for in the model? Do you have any evidence that some of these might be detectable, or even significant?

Conclusion

What is the moment of inertia of the rotor? Does the kinematic model you developed for the system adequately describe the behavior of the system?