Worksheet 3: Uniform Circular Motion

Problem

Just as trajectory motion can be expressed by two one-dimensional position-time equations $x = x_0 + v_{0x}t$ and $y = y_0 + v_{0y}t - \frac{1}{2} gt^2$, uniform circular motion can be expresed by the two one-dimensional position-time equations

$$x = r\cos(\omega t)$$
and
$$y = r\sin(\omega t).$$

a. What is the physical meaning of r?

b. What are the units of r?

c. What is the physical meaning of ω?

d. What are the units of ω?

e. The position vector $\vec{r}$ can be expressed as $\vec{r} = (x, y)$; likewise, the velocity vector $\vec{v}$ can be expressed as (v_x, v_y), where the x- and y-components v_x and v_y are $v_x = dx/dt$ and $v_y = dy/dt$.

  (i) What is dx/dt?

  (ii) What is dy/dt?

f. What is v, the magnitude of the velocity vector $\vec{v}$? This is the speed of the object. Use the Theorem of Pythagoras to find the formula and simplify.

g. What are the units of v_x, v_y, and v?

h. How does the direction of the vector $\vec{v}$ compare to the direction of the vector $\vec{r}$? (*Hint:* $\cos(\theta) = \sin(\theta+\pi/2)$, $\sin(\theta) = \cos(\theta-\pi/2)$, $-\cos(\theta) = \sin(\theta-\pi/2)$, and $-\sin(\theta) = \cos(\theta+\pi/2)$.)

i. The acceleration vector $\vec{a}$ can be expressed as $\vec{a} = (a_x, a_y)$, where $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

  (i). What is dv_x/dt?

  (ii) What is dv_y/dt?

j. What is a, the magnitude of the acceleration vector $\vec{a}$? Use the Theorem of Pythagoras to find the formula and simplify.

k. What are the units of a, a_x, and a_y?

l. Using the formula for v that you found in part f, find the formula for a in terms of v rather than ω. What is it?

m. What is the direction of the vector $\vec{a}$ compared to the direction of the vector $\vec{v}$?

n. What is the direction of the vector $\vec{a}$ compared to the direction of the vector $\vec{r}$?