Name: ____

LAB 12. SIMPLE PENDULUM

Introduction

A pendulum experiences a restoring force when displaced from its straight-down equilibrium position. If the displacement angle of the pendulum is small, the force is almost directly proportional to the displacement angle.

In this lab you will examine how closely the behavior of a pendulum matches the behavior predicted for a simple, approximate model.

Theory

Simple pendulum

The tangential component of force acting on the center of gravity of a pendulum is $-mg\sin\theta$, where m is the mass of the bob and θ is the angular displacement. The torque about the pivot point is $\tau = -Lmg\sin\theta$, where L is the length of the pendulum. If the bob is a point mass, the moment of inertia of the pendulum is mL^2. Although it cannot actually be a point mass, practically mL^2 is a good approximation if L is much larger than the size of the bob.

If the displacement angle θ is small, then by the small-angle approximation $\sin\theta \approx \theta$, with θ expressed in radians. We therefore approximate $\tau \approx -Lmg\theta$, which matches Hooke's law for a torsional oscillator $\tau = -\kappa\theta$; here, $\kappa = Lmg$.

For the same reasons that the angular frequency of a linear Hooke's law oscillator obeys $\omega^2 = k/m$, for a torsional oscillator $\omega^2 = \kappa/I$. For this pendulum, this is $Lmg/(mL^2) = g/L$.so $\omega 2 = g/L$. Since the period T is related to the angular frequency ω as

$$T = 2\pi/\omega$$

$$T^2 = 4\pi^2\omega^2$$

$$T^2 = 4\pi^2 L/g. \qquad (1)$$

Thus a plot of T^2 vs L should give a straight line with a slope of $4\pi^2/g$ passing through the origin.

Fitting a direct proportion

A model predicts measurements of an observed variable y as a function $y = f(x)$ of input variables x. The "least squares" gauge of how well the model fits that data is the fit score

$$S^2 = \sum_{i-1}^{N}\left(y_i - f(x_i)\right)^2$$

For a direct proportion model $f(x) = Ax$, the value of A minimizing S^2 is given by the formula

$$A = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2} \qquad (2)$$

In this model, $y = T^2$, $x = L$, and $A = 4\pi^2/g$. Therefore, the best-fit A calculated using (2) allows an estimate of the local gravitational field by solving $g = 4\pi^2/A$.

Equipment

Clamp stand, bar with protractor, table clamp, thread, 10-g, 20-g, 50-g, and 100-g disk weights, timer, meter stick

Activities

Make the pendulum by threading the thread through the hole in the center of a disk weight. Measure the length of the pendulum from the pivot to the center of mass of the disk weight. To measure the period of the pendulum, start it in motion, then begin timing after several cycles have transpired. Time at least ten cycles and divide the time t by the number of cycles N to find the period T.

Keep the pivot of the pendulum as motionless as possible. Make sure that the rod is screwed tightly into the base, and clamp the base to the table. Tighten the clamp holding the crossbar onto the rod. Watch for signs of slipping and correct as soon as possible.

Mass and length

Use a small initial amplitude, large enough to easily see at least ten cycles. Measure the period of oscillation at five different pendulum lengths for each of the four different weights. The largest length should be the longest you can experimentally manage; the shortest should be no less than ten times the radius of the disk weight. Try to make the pendulum lengths the same for the different weights. It will be impossible to match them exactly, however, so measure the pendulum length for each different weight-length combination.

10 g		20 g		50 g		100 g	
L	T	L	T	L	T	L	T

Amplitude

Choose an intermediate pendulum length L and use the 100-g disk weight. Measure the period of oscillation from initial amplitudes Θ ranging from 10° up to 90° in ten-degree increments. Using the pendulum as a plumb line, position the protractor to orient 270° vertically downward. Check that it maintains the proper orientation before each trial.

Pendulum length L = _____________

Θ	T	Θ	T	Θ	T
10°		40°		70°	
20°		50°		80°	
30°		60°		90°	

Data Processing

Mass and length

For each weight:

1. Plot T^2 (vertical axis) vs L (horizontal axis).
2. Find the constant of proportionality A using formula (2).
3. Calculate the gravitational field $g = 4\pi^2/A$.
4. Plot the residuals $T_i^2 - AL_i$ (vertical axis) vs L (horizontal axis).

Amplitude

1. Calculate the mean value $\bar{T}$ of the period from all the trials.
2. Plot the residuals $T_i - \bar{T}$ (vertical axis) vs Θ (horizontal axis).

Lab Report

Present your findings in a brief, lucid report. It should contain the following parts.

Mass and length

Results

For each weight:

- Show the data.
- Show the plot of T^2 vs L.
- Show the plot of residuals vs L.
- Report A and g from each weight.

Discussion

Do your data show that the model satisfactorily predicts the period of these pendulums? Explain your conclusion. Discuss the residuals plots in your explanation.

Amplitude

Results

Show the data. Show the plot of residuals vs. Θ.

Discussion

Do your data detect a departure from the simple pendulum model at large displacement angles? Explain your conclusion. Discuss the residuals plot in your explanation.