

LAB 10. TORQUE AND MOMENT OF INERTIA

Introduction

In this activity you use a falling mass to pull a string, which generates a torque to accelerate a heavy rotor with an unknown moment of inertia. You measure the acceleration of the falling mass to determine the rotor's moment of inertia.

Apparatus

The rotor is mounted in the bearing with its axis vertical. Its axle is fixed to three pulleys of different radius. One end of a string is wound around one of the pulleys, so that pulling the string horizontally away from the pulley produces a torque about the axis. The string runs over a "smart" pulley and the other end is attached to a hanging mass, so that the weight of the hanging mass pulls on the string and the string in turn torques the rotor.

The "smart" pulley is equipped with a photogate sensor, allowing the PASCO Capstone software to measure the speed of the string running over it.

Theory

As the hanging mass falls, it turns the pulley attached to the axle of the rotor. When the hanging mass falls a distance d , the pulley, of radius r , advances an angle $\theta = d/r$ radians.

Correspondingly, when the mass falls at speed v , the pulley rotates at angular speed $\omega = v/r$; if the acceleration of the falling mass is a , the angular acceleration of the pulley is $\alpha = a/r$.

The only force promoting the descent of the mass is its gravitational attraction mg to the earth. Opposing the acceleration is its mass and the rotational inertia (moment of inertia I) of the rotor. The tension T in the string linking the rotor and the hanging mass determines their acceleration: the net force on the hanging mass is a downward $\Sigma F = mg - T$, and the only torque on the pulley of the rotor is $\Sigma \tau = rT$. The acceleration of the hanging mass is then $a = \Sigma F/m$, and the angular acceleration of the rotor is $\alpha = \Sigma \tau/I$.

Experiment

In this activity you will pull on the string with three different hanging masses, and wind the string around the three different pulleys on the rotor axle. In each of the nine cases, you will measure the acceleration of the hanging mass.

Supplies

Rotor apparatus, string, photogate pulley, PASCO interface and computer with Capstone software installed, 100-g, 200-g, and 500-g hanging masses, Vernier calipers, ruler, floor pad.

Data Collection (by group)

Setup

1. Measure the mass and radius of the rotor. Mass = _____ kg; Radius = _____ m
2. Use the Vernier calipers to measure the diameters of each of the three pulleys on the rotor axle.
3. Connect the smart pulley to the PASCO interface and recognize the sensor in Capstone.
4. Mount the smart pulley to the edge of the table and place a pad under it.

Measurement

1. Wind the string around one of the pulleys, leaving about 20 cm free.
2. Mount the rotor into the bearing. Run the string over the smart pulley and hang a mass at the end of the string. Hold the rotor still.
3. Start data collection.
4. Release the rotor. When the falling mass reaches the floor, stop the spinning rotor and stop data collection.
5. Make a velocity-time plot of the data in Capstone. Fit the linear portion of the plot with a linear trend line. The slope of this trend line is the acceleration of the falling mass.
6. Record the hanging mass, the pulley radius, and the acceleration.
7. Repeat the run. If the two accelerations are not within 5% of each other, measure a third time.
8. Measure the acceleration of each of the three masses using each of the three pulleys on the rotor.

Measurements

m (kg)	r (m)	a (m/s ²)

Data Processing (individual)

Theory

Solve the equations above to find the formula for acceleration in terms of m , r , and I .

Data Processing

1. Open the Sheets spreadsheet provided for the lab. Make a copy to edit for your own use.
2. Enter your measurements of m , r , and a for each trial into the pink cells. If you need to add more rows, be sure to “fill down” the formulas in the columns to the right.
3. Examine the values of I calculated in the spreadsheet from each acceleration. Pick a first estimate of I that seems mostly consistent with them.
4. Enter this first estimate of I into the spreadsheet, in the “Trial I ” cell. The spreadsheet will calculate expected accelerations a_{calc} using that value.
5. Calculate the “residual” s , the difference between the predicted and observed accelerations, for each run using the formula $s_i = a_i - a_{ci}$.
6. Calculate the “fit score” SSR as the sum of squared residuals, $SSR = \sum_{i=1}^N s_i^2$.
7. Experiment with different values of I to find the one giving the smallest SSR . This is the best-fit estimate of I . Record the values of I you tried, along with their resulting SSR values. (There is a place in the spreadsheet for these records.)
8. Make a plot of the residuals (vertical axis) vs. mr^2 (horizontal axis).
9. Make a plot of the SSR (vertical axis) vs. I (horizontal axis).
10. Calculate what the moment of inertia should be for a uniform cylinder with the mass and radius of the rotor.

Lab Report (individual)

Present your findings in a brief, lucid report. It should contain the following parts.

Data

Show the raw data tables, reporting m , r , and a for each run.

Theory

Show the derivation of the formula for a as a function of m , r , and I .

Results

Report the best-fit estimate of I . (Don't forget the units!)

Report the value of I calculated from the measurements of the rotor.

Show the graphs that you made.

- The residuals plot from the best estimate of I
- The plot of fit score SSR vs I value.

Discussion

Do the predictions from the model using your best-fit estimate of I adequately match the observed accelerations? If yes, explain what constitutes “adequately”. If no, suggest how the model could be modified to more faithfully fit the data.