Name: ______________________________________

LAB 10. ROLLING DOWNHILL

Introduction

A rolling object has rotational as well as translational kinetic energy. As it rolls down an incline, its gravitational potential energy converts to kinetic energy. The distribution of kinetic energy between translational and rotational forms depends on the object's moment of inertia.

The center-of-mass moment of inertia I of a round object rotating about its central axis is usually written in the form $I = cMR^2$, where M is the object's mass, R is its outer radius, and c is a number depending on the distribution of the mass in the object. The object's rotational kinetic energy is $\frac{1}{2} I\omega^2$, where ω is the object's rotational velocity.

If the object rolls, its rotational velocity ω and translational velocity v are related as $v = \omega R$. Its total kinetic energy is the sum of the contributions from translation and rotation:

$$K_{tot} = K_{trans} + K_{rot} = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2 \quad (1)$$

Substituting cMR^2 for I and v/R for ω allows the translational and rotational terms to be combined.

$$K_{tot} = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} cMR^2(v/R)^2$$

$$K_{tot} = (1 + c)\ \tfrac{1}{2} Mv^2 \quad (2)$$

When the object rolls from rest a height h down an incline with conservation of mechanical energy, its initial gravitational potential energy becomes kinetic energy.

$$Mgh = (1 + c)\ \tfrac{1}{2} Mv^2 \quad (3)$$

If the slope of the ramp is constant, its downhill acceleration also will be constant and the distance x traveled is given by

$$x = \tfrac{1}{2} (v_0 + v)t \quad (4)$$

where v_0 is the initial velocity (starting from rest, $v_0 = 0$), v is the final velocity, and t is the travel time. Measuring the travel time t and distance x allow us to find the final speed v. Then we can solve equation (3) to find the coefficient c.

Alternatively, if you know the acceleration of the rolling object, you can determine c from it. The angular acceleration of the rolling object about its center of mass is $\alpha = \Sigma\tau/I$, where the torques τ are calculated about the object's center of mass. The only force giving a torque is static friction f; $\tau_f = Rf$, so $\alpha = Rf/I$. The downhill acceleration is

$$a = (Mg \sin\theta - f)/M \quad (5)$$

where θ is the angle of the incline below horizontal and f is static friction. When the object rolls without slipping,

$$a = R\alpha = R^2 f/I = R^2 f/(cMR^2)$$

$$a = f/(cM) \quad (6)$$

From equations (5) and (6) we can find f and c.

In this activity we will compare the coefficients *c* we estimate from our measurements to the theoretical values from the moment of inertia formulas. Finally, we will empirically determine the *c* for an object with a complicated or unknown mass distribution, and compare it to that of known shapes to tell us something about its composition.

Supplies

Stopwatch, ruler, meter stick, motion sensor apparatus, Vernier calipers, sphere, hoop, cylinder, straight ramp, pad

Data Collection

Setup

1. Elevate one end of the ramp.
2. Place a pad at the low end of the ramp so that the rolling objects don't slam into the bench top.
3. Measure and record the elevation of the starting position above the end of the ramp. This is the height h.
4. Make measurements on the following objects: a solid cylinder, a solid sphere, a hoop, a hollow cylinder, and an irregular or unknown object. Measure and record all pertinent dimensional information for each object: shape, outer radius R, thickness (if a hoop), length along the axis, etc.

Object	Measurements

Velocity Measurement

1. Measure the height y of some position on the ramp above the table top. Record the position s as well. The sine of the angle of the ramp is then $\sin\theta = y/s$.

   y: _____________; s: _____________

2 Place the motion sensor at the top of the ramp.

3. Hold the object in front of the sensor.
4. Start data collection.

5. Make a velocity-time graph of the data.
6. If the graph is credible, fit a linear equation to the linear portion of the graph. The slope of this graph is the acceleration.
7. Record the acceleration from three credible runs for each rolling object.

Object	Acceleration, m/s^2		
	Run 1	Run 2	Run 3

Travel Time Measurement

1. Measure and record the distance on the ramp from where you will start the rolling object to the end. This is the travel distance *x*.

   x: ______________

3. Place the object at the top of the ramp. Hold it in place by a piece of cardboard or wood in front of it.
4. To start the object rolling down the hill, quickly pull the cardboard or wood downhill away from the object. At exactly the same time, start the stopwatch.
5. Stop the stopwatch when the object reaches the end position of the ramp. Discard the run if the object hits the edge of the track or otherwise becomes unreliable.
6. Measure and record five travel times for each object.

Object	Travel time, s				
	Run 1	Run 2	Run 3	Run 4	Run 5

Data Processing

Theory

1. Solve equation (4) above to obtain the formula for *v* in terms of *x* and *t*.
2. Solve equation (3) above to obtain the formula for *c* in terms of *v* and *h*.

3. Solve equations (5) and (6) to obtain the formula for c in terms of a, y, and s.

Velocity data

1. Calculate the mean acceleration of each object.
2. Use this mean value to estimate c for each object.

Travel time data

1. Calculate the mean $\bar{t}$ and standard deviation σ_t of travel times for each object.
2. Use $\bar{t}$ and x to estimate the final speed v and coefficient c for each object.
3. Repeat each estimation of v and c using $\bar{t} - 2\sigma_t$ and $\bar{t} + 2\sigma_t$ to obtain confidence intervals for the estimates.

Lab Report

Present your findings in a brief, lucid report. You don't need a spreadsheet, but a spreadsheet may be useful. The report should contain the following parts.

Data

Show the raw data tables.

Theory

Show your work to derive the formulas for v and c in terms of x, t, and h. Show your work to obtain the formula for c in terms of a, y, and s.

Results

For each object that you rolled, tell me a, $\bar{t}$, σ_t, and your estimates and confidence intervals for v and c.

Discussion

For each known object,

- Does your empirical c obtained from acceleration match the empirical c obtained from travel time?
- Do the different empirical values of c match the theoretical c for the shape?
- Are your measurements accurate and precise enough to distinguish the c of one shape from another? Justify your answer.

For the unknown or irregular object, what does the c tell you about its mass distribution? Explain.