Name: ______________________________

LAB 12. HOOKE'S LAW

Introduction

External force applied to an object will change the object's size or shape. Objects exhibiting elastic behavior will return to their original shapes when the stress is removed. The simplest approximation to this behavior is Hooke's law, $F = -kx$, where F is the force exerted by the object, k is its stiffness, and x is its distortion from its equilibrium size.

The equation governing the motion of a mass m acted on only by a Hooke's law spring with force constant k is $m\, d^2x/dt^2 = -kx$. The function of time t that is the general solution of this differential equation is $x = A \cos(\omega t + \phi)$, where $\omega^2 = k/m$ and ϕ is a phase offset. Note that the oscillation frequency $f = 2\pi\omega$ does not depend on the oscillation amplitude A.

In this lab, you will determine if Hooke's law adequately models the behavior of several springs. You will further test the claim that $\omega^2 = k/m$, and that amplitude does not affect oscillation frequency.

Lab activity

Supplies

Three springs, several weights, meter stick, rod and table clamp, stopwatch or timer

Procedure

1. Measure and record the initial length l_0 of the spring.
2. Hang a known mass from the spring and allow the mass and spring to equilibrate. Record the spring's length l_1.
3. Set the mass into vertical oscillations. Time a number N (say, 10–20) of complete oscillations. Record N and the total time T_N. Divide T_N by N to find the period. For a few of these, repeat with a different oscillation amplitude to check if the period of oscillation changes.
4. Add more known masses incrementally until you have seven (7) mass, length, and period measurements in addition to the zero-load length. Record these additional data as well.
5. Convert the masses to the forces they exert on the spring by multiplying by the gravitational field g. In other words, $F = mg$. Record these forces.
6. Repeat the process with other springs, for a total of at least three springs.

Data Processing

You now have data sets of tension, length, mass, and oscillation period for several springs. From these data, estimate the spring constant k of each spring in two different ways: from the equilibrium lengths measured at each tension, and from the oscillation periods measured at each mass. Also use the data to evaluate the model claims that the length and tension of the spring are related by Hooke's law, and that the motion of the system is a sinusoid with angular frequency $\omega = \sqrt{k/m}$.

Spring constant from tension

For each spring, you made a set of measurements of (F, L), where F is tension (mg) and L is spring length. If Hooke's law is a good description of the spring's behavior, $F = k(L–L_0)$, where L_0 is the length of the spring without tension. A little algebra yields $L = F/k + L_0$, which means that a plot of L vs F should give a straight line with a slope of $1/k$ and a y-intercept of L_0. If the plot gives a straight line, you can conclude that Hooke's law is an adequate model for the spring's behavior for the tensions you studied. Then you can fit a least squares straight line ($y = Ax + B$) to the plot, and from the slope and y-intercept of the line deduce the spring's spring constant k and resting length.

Spring constant from period

For each spring, you made a set of measurements of (m, T), where m is mass of a hanging weight and T is its period of oscillation on the spring. If Hooke's law is a good description of the spring's behavior, $\omega^2 = k/m$, where $\omega = 2\pi/T$ is the angular frequency of oscillation. A little algebra yields $T^2 = 4\pi^2 m/k$, which means that a plot of T^2 vs m should give a straight line with a slope of $4\pi^2/k$ that passes through the origin. If the plot fits those criteria, you can fit it with a least squares direct proportion model ($y = Ax$), and from its slope deduce the spring constant k.

Lab Report

Sure, this can be a group report.

Abstract

Briefly describe the system. Identify the quantities that were measured, and the quantities that were inferred from the measurements.

Purpose

What are these measurements used for? To what end are they the means?

Theory

A lot of the important theory was presented in class and in the introduction to these instructions. Explain here. Also derive from the model how the measured quantities are expected to behave.

Experimental

Describe the experimental apparatus and measuring equipment. Give the procedure you followed to make the measurements.

Observations and Data

You don't need to transcribe the data directly into a lab report, but you should clearly communicate what you found. If you choose to graph your data, it would make sense to use a spreadsheet. If your spreadsheet is online, you can link to it in your report.

Analysis and Discussion

What did the Hooke's law model predict for your observations? Did your observations conform to these predictions? Use the data to answer these questions.

Conclusion

Dos Hooke's law adequately describe the behavior of the springs you studied in this experiment?