Name: ____

LAB 11. ROLLING DOWNHILL

Introduction

A rolling object has rotational as well as translational kinetic energy. As it rolls down an incline, its gravitational potential energy converts to kinetic energy. The partition of kinetic energy between translational and rotational forms depends on the object's moment of inertia.

The rotational kinetic energy is $\frac{1}{2} I\omega^2$, where ω is the object's angular speed. Its center-of-mass moment of inertia I is usually written in the form $I = cMR^2$, where M is the object's mass, R is its outer radius, and c is a number depending on the distribution of the mass about the axis.

Conservation of Mechanical Energy

If the object rolls without slipping, its rotational speed ω and translational speed v are related as $v = \omega R$. Its total kinetic energy is the sum of the contributions from translation and rotation:

$$K = K_{tr} + K_{rot} = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2 \tag{1}$$

Substituting cMR^2 for I and v/R for ω allows us to combine the translational and rotational terms.

$$K = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} cMR^2(v/R)^2$$

$$K = (1 + c)\,\tfrac{1}{2} Mv^2 \tag{2}$$

When the object rolls a height h down an incline with conservation of mechanical energy, its decrease in gravitational potential energy Mgh becomes kinetic energy.

$$Mgh = (1 + c)\,\tfrac{1}{2} Mv^2 \tag{3}$$

If the slope of the ramp is constant, its downhill acceleration also will be constant and the distance s traveled is given by

$$s = \tfrac{1}{2} (v_0 + v)t \tag{4}$$

where v_0 is the initial velocity (if starting from rest, $v_0 = 0$), v is the final velocity, and t is the travel time. Measuring the travel time t and distance s allow us to find the final speed v. Then we can solve equation (3) to find the coefficient c.

Kinematics

Alternatively, if we measure the downhill acceleration a of the rolling object, we can determine c from it. The angular acceleration of the rolling object about its center of mass is $\alpha = \Sigma\tau/I$. The only force giving a torque is static friction f; $\tau_f = Rf$, so the static friction determines the angular acceleration.

$$\alpha = Rf/I \tag{5}$$

The downhill acceleration is

$$a = (Mg \sin\theta - f)/M \tag{6}$$

where θ is the angle of the incline below horizontal. When the object rolls without slipping,

$$a = R\alpha = R^2f/I = R^2f/(cMR^2)$$

$$a = f/(cM) \tag{7}$$

Equations (6) and (7) give the quantities a, f, and c in terms of each other. Given one quantity, the two equations can simultaneously yield the other two quantities. In our case, we will measure the downhill acceleration a; thus we can find f and c. We are interested in c.

In this activity we will compare the coefficients c that we obtain from our measurements to the theoretical values from the moment of inertia formulas. Finally, we will empirically determine the c for an object with a complicated or unknown mass distribution, and compare it to that of known shapes to tell us something about its composition.

Supplies

Stopwatch, tape, ruler, meter stick, motion sensor apparatus, Vernier calipers, sphere, hoop, cylinder, straight ramp, pad

Data Collection

Setup

1. Elevate one end of the ramp.
2. Place a pad at the low end of the ramp so that the rolling objects don't slam into the bench top. Prepare to catch them before they fall to the floor.
3. Make measurements on the following objects: a solid cylinder, a solid sphere, a hoop, a hollow cylinder, and an irregular or unknown object. Measure and record all pertinent dimensional information for each object: shape, outer radius R, thickness (if a hoop), length along the axis, etc.

Object	Measurements

Velocity Measurement

1. Measure the height difference h between any two points on the ramp. Record the distance s along the ramp between those two points as well. (If you use the starting and ending positions from the Travel Time part of this activity, you only need to measure these once.) The sine of the angle of the ramp is then $\sin\theta = h/s$.

   h: ______________; s: ______________

2 Place the motion sensor at the top of the ramp.

3. Hold the object in front of the sensor.
4. Start data collection. Release the object so that it rolls down the ramp, away from the sensor.
5. Make a velocity-time graph of the data.
6. If the graph is credible, fit a linear equation to the linear portion of the graph. The slope of this portion is the acceleration.
7. Record the acceleration from three credible runs for each rolling object.

Object	Acceleration, m/s^2		
	Run 1	Run 2	Run 3

Travel Time Measurement

1. Measure and record the elevation of the starting position above the end. This is the height *h*. Measure and record the distance on the ramp from where you will start the rolling object to the end. This is the travel distance *s*.

   h: _____________; *s*: _____________
2. Place the object at the starting position. Hold it in place by a piece of cardboard or wood in front of it.
3. To start the object rolling down the hill, quickly pull the cardboard or wood downhill away from the object. (You are trying not to push it). At that moment, start the stopwatch.
4. Stop the stopwatch when the object reaches the end position of the ramp. Discard the run if the object hits the edge of the track or anything else goes wrong.
5. Measure and record five travel times for each object.

Object	Travel time, s				
	Run 1	Run 2	Run 3	Run 4	Run 5

Data Processing

Theory

1. Solve equation (4) above to obtain the formula for v in terms of s and t.
2. Solve equation (3) above to obtain the formula for c in terms of v and h.
3. Simultaneously solve equations (6) and (7) to obtain the formula for c in terms of a, h, and s.

Velocity data

1. Calculate the mean of the accelerations of each object.
2. Use the mean acceleration for each object to estimate c for the object.

Travel time data

1. Calculate the mean $\bar{t}$ and standard deviation $\sigma_t = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(t_i - \bar{t})^2}$ of travel times for each object.
2. Use $\bar{t}$ and s to estimate the final speed v and coefficient c for each object.
3. Repeat each estimation of v and c using $\bar{t} - 2\sigma_t$ and $\bar{t} + 2\sigma_t$ in place of $\bar{t}$ to obtain confidence intervals (high and low bounds) for the estimates of v and c.

Lab Report

Present your findings in a brief, lucid report. You don't *need* a spreadsheet, but a spreadsheet will probably be useful.

Theory

Show your work to derive the formulas for v and c in terms of s, t, and h; and for c in terms of a, h, and s. (It is probably easiest to hand write this, unless you want practice typesetting equations.)

Data

Show the raw data tables.

Analysis

For each object that you rolled, tell me a, $\bar{t}$, σ_t, and your estimates and confidence limits for v and c.

Discussion

For each known object,

- Does your empirical c obtained from acceleration match the empirical c obtained from travel time?
- Do the different empirical values of c match the theoretical c for the shape?
- Are your measurements accurate and precise enough to distinguish the c of one shape from another? Justify your answer.

For the unknown or irregular object, what does the c tell you about its mass distribution? Explain.