LAB 6. WORK-ENERGY THEOREM

Introduction

Un this lab, you will test Hooke's law and the work-energy theorem. A force F will be applied to a moving dynamics cart as you measure the cart's displacement s and velocity v. You will compare the work done on the cart $W = \int_{s_1}^{s_2} F ds$ to its change in kinetic energy $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$.

Supplies

Both activities: dynamics cart with accessory masses, track, motion detector apparatus, hanging mass set

Hooke: spring, anchor for spring, meter stick

Gravity: pulley or smart pulley, level, string, scissors

Activities

Hooke's law force

In this activity, you will study and use the changing force of a spring. The magnitude of the force depends on the elongation of the spring. First, you will test the spring to see if its force follows Hooke's law. If it does, you will determine a quantitative formula for its tension as a function of length. Next, test the work-energy theorem by measuring the cart's position and speed as the spring pulls it. You will use the quantitative model to calculate the work done along its displacement, and the formula ½ mv^2 to calculate its kinetic energy.

Characterize the spring

Collect the data

To find the relationship between tension and length of the spring, hang the spring from a support. Measure and record its length by itself, and when supporting a variety of hanging masses. Record your data on your own paper.

Process the data

You measured the length of the spring as it supported different masses. Convert the masses m into spring tensions $F_T = mg$. The Hooke's law model predicts that the length L of the spring will be a linear function of the tension mg, from $mg = -k(L-L_0)$. So, a plot of L vs. m ought to give a straight line. If it does, Hooke's law is valid for this spring for the tensions you applied. If it doesn't, then you have an empirical tension-extension relation that you can use.

Pull the cart

Collect the data

Measure and record the mass of your cart and of any auxiliary masses it carries. Secure one end of the spring near the far end of your track. Fasten the cart to the other end of the spring. Place the motion detector at the opposite end of the track from the spring. Position the cart on the track so that the spring pulls on it. Record the spring length at the initial position of the cart. Start data collection. Release the cart so that it is pulled along the track by the spring. Catch the cart before it runs into anything. Your data should show the position and speed of the cart at each measurement time.

Process the data

You need to find the length of the spring at each measurement time. You know the length of the spring L_1 at the beginning of the run, and you know the position s_1 of the cart at the same time. You can use the fact that L and s change together to find L at any time using $L = L_1 – \Delta s$. Disregard data for which the spring is shorter than its equilibrium ($F_T = 0$) length.

Because you can find the length of the spring at each time, you can determine how much work the spring has done on the cart by each time. Because you measured the mass of the cart and its speeds at the different times, you can calculate its kinetic energy at each time. Beware, however, that Logger Pro averages positions to smooth its calculations of velocity. For that reason, its reported values of velocity are not reliable when velocity changes abruptly, such as right after the cart is released.

The work-energy theorem predicts that the work done on the cart by all the forces acting on it equals its change in kinetic energy. Do your data and calculations show this?

Gravitational force

In this activity, the cart on a level track is pulled forward by a weight hanging from a cord running over a pulley. Work is done on the cart along its displacement by the tension of the cord.

Collect the data

Measure and record the mass of the cart with any auxiliary masses and the mass of the hanging weight. Set up the apparatus so that the motion detector observes the position and velocity of the cart as the falling weight pulls the cart away from the detector. Begin data collection. Release the cart to be pulled along the track by the falling weight. Catch the cart before it hits the pulley at the end of the track. Repeat with a variety of hanging masses and, if time permits, with a variety of auxiliary masses on the cart.

Find the position of the cart when the hanging weight reaches the floor. Only before this position will the hanging weight pull on the cart, so only measurements before this position are meaningful for this investigation.

Process the data

As the cart moves forward a distance Δs, the hanging weight falls the same distance. Calculate the total work done on the cart and weight from the change in gravitational potential energy $–mg\Delta s$, and calculate the kinetic energy of the cart and hanging weight from the measured speed.

Before You Leave

Show your instructor the data you collected. It should include

- **Hanging spring**: supported mass and length of the hanging spring
- **Spring-pulled cart**: mass of cart + auxiliary mass, initial spring length, several positions and speeds of cart while pulled by the spring
- **Weight-pulled cart**: mass of cart + auxiliary mass, mass of hanging weight, several positions and speeds of cart while pulled by the hanging weight

When you need to record data of position and simultaneous speed from the motion sensor apparatus, it may be easiest to have Logger Pro “export” the data as a csv or other type of tabular file. You can save it onto a thumb drive, store it on the cloud, or email it to yourself.

Data Analysis

A spreadsheet will make this easiest. Doing the calculations by hand would be torture.

Hanging Spring

Have the spreadsheet make a plot of spring length vs. supported mass. If the plot is linear, then Hooke's law adequately models the data.

How can you assess if the plot is linear? How can you assess if it is not linear?

Hooke's law predicts that $F = -kx$, where k is a characteristic of a particular spring. How can you find k from your data? Do you need to know what x is?

Determine the formula to predict the tension applied by the spring at different lengths of the spring.

Spring-pulled cart

First, determine which (t, s, v) data points were collected while the spring was pulling on the cart with a Hooke's law force. From s, find the potential energy U of the spring; from v, find the kinetic energy K of the cart. The work done by the spring on the cart at any position is $U_1 - U$, where U_1 is the spring's potential energy where the cart was released and U is its potential energy at that particular position.

Weight-pulled cart

The work done on the cart-weight system is the work done by gravity on the weight as it descends. When calculating the kinetic energy of the system, include the mass of the dropping weight as well as the mass of the cart; they both move at the same speed. They don't move at the same direction, but that does not matter for kinetic energy.

Lab Report

This can be a group report.

Abstract

What systems did you study? What quantities did you measure, what quantities did you calculate from the measurements, and what idea did you test?

Purpose

What aspect of physics was this intended to illuminate? What experimental skills did this teach?

Theory

Spring force

What does Hooke's law predict about the force-length relationship of a spring? What does Hoke's law predict that you will observe when measuring the length of a spring supporting different masses? How can you evaluate if Hooke's law adequately models a spring's behavior? How can you find the spring constant k from the data?

Spring-pulled cart

The kinematic measurements in the experiment are (t, s, v). How do you determine the spring length L, its distortion x, and its potential energy U at each time? How do you determine the work done on the cart by the spring at each position?

Weight-pulled cart

How do you calculate the tension in the cord while the falling weight pulls the cart? The kinematic measurements in the experiment are (t, s, v). How do you determine the work done on the cart by the tension in the cord at each position?

Experimental

In a few sentences, describe the apparatus for each experiment, what you did with it, and the measurements you took. Identify commercial products by manufacturer and model.

Observations and Data

Show your raw data tables. You may scan them or transcribe them into a spreadsheet. Clearly indicate the primary data in the spreadsheet.

Analysis and Discussion

Consider and discuss how well the model fits the data, suggesting reasons for how well or how poorly it fits.

Hanging spring

Present a plot of the observed length vs. supported mass data. Identify the quantities (spring constant k, equilibrium length L_0) derived from these data. On the same axes, plot a smooth curve of predictions of the Hooke's law model. Discuss how well or poorly the predictions match the observations, and explain if you can.

Spring-pulled cart

You ran the experiment several times. For a few runs that you believe are reliable, plot the calculated kinetic energy of the cart vs. position. On the same axes, plot the calculated work done by the spring on the cart. Discuss how well or poorly the two curves coincide, and explain if you can.

Weight-pulled cart

For a few runs that you believe the measurements are reliable, plot the calculated kinetic energy of the cart vs. position. On the same axes, plot the calculated work done by the tension of the cord on the cart. Discuss how well or poorly the two curves coincide, and explain if you can.

Conclusions

Is the spring accurately modeled by Hooke's law? Is the change in the cart's kinetic energy from one position to another equal to the total work done on the cart between those positions? Identify the evidence supporting your conclusion.