## LAB 3. PROJECTILE RANGE

## Approach

You and your group will decide what measurements to take and what analyses to perform to answer the investigation question. Before you begin, decide what measurements you will need and how to take them. Communicate your plan to the instructor or a TA before taking any measurements. Your experimental plan must be approved before you begin for your lab to receive credit. Before you leave the lab, show your data to the instructor or a TA to verify that it is complete, sufficient, and realistic. Your data must be approved before you leave for your lab to receive credit.

## Investigation

A spring-powered launcher clamped on a lab table fires a one-inch steel ball, which lands on the floor. You will experimentally find how changing the launch angle affects the distance traveled by the ball before it strikes the ground. Of particular interest is finding the launch angle that gives the ball its greatest range.

You also will make a mathematical model of the projectile's trajectory, and use that model to predict the angle of greatest range. You will compare the model's predictions to your data.

## Questions to ponder

What are the equations of motion for the trajectory of a launched projectile?
What launch angle gives the greatest range for a projectile launched from a lab table onto the floor?

## Tasks

- Find the muzzle speed.
- Find how consistent the muzzle speed is.
- Measure the launch height.
- Mathematically model the projectile's range given launch height, speed, and angle.
- Measure the range resulting from a variety of launch angles.
- Record your primary data in your lab notebook in a manner clear enough for a reader to understand what you did and what you found.
- Find the launch angle giving greatest range.
- Use the mathematical model to predict the greatest range and the angle producing it.
- Compare the data to the model.


## Operating the Launcher

## Description

The launcher propels its projectile using a cocked spring. The spring can be set at three different starting compressions. The barrel pivots up and down to adjust the firing angle. An angle gauge with a small built-in plumb line measures the launch angle. The muzzle is the end of the barrel close to the pivot, so that changing the launch angle does not change the launch height.

In this activity you will clamp the launcher to a table and fire a projectile onto the floor from different initial angles. You will then compare the observed trajectories to predictions from a simple ballistic model.

## Supplies

Projectile launcher, projectile, ramrod, clamp, plumb line, measuring tape, meter stick, carbon paper, white paper, tape, sturdy table, safety goggles or glasses

## Safety considerations

- The projectile launcher fires a $1^{\prime \prime}$ steel or hard plastic ball at speeds that can be considerable. To avoid eye injury, everyone in the room must wear safety glasses or goggles while launchers are in use.
- Never look down the muzzle of barrel. You can see into the barrel through the slots cut in the sides.
- Once the barrel is loaded, be careful not to place any part of your (or anyone's!) body in front of the launcher.
- Use the ramrod for loading and cocking the launcher. Never poke your fingers into the launcher-you could very easily break them! That would be bad.
- Load the projectile launcher with $1^{\prime \prime}$ balls only. Other loads may cause dangerous conditions.


## Setup

Place the launcher at the edge of the table, with the muzzle of the barrel pointing away from the table. Clear the line of fire so that the trajectory is not obstructed.
Clamp the launcher firmly to the table. Use two clamps, one on either side of the base, if available. If only one clamp is available, put it on the side under the barrel of the launcher.

## Firing

Place the projectile into the muzzle of the launcher. Push it into the barrel using the ramrod. As you push it in, the trigger on the top of the barrel will rise and fall with a click up to three times. After each click, the launcher is cocked in that position.
If the barrel is horizontal, check that the projectile is against the cradle by looking through the slots in the side of the barrel. If the projectile has rolled forward, gently push it back against the cradle with the ramrod.

Check the launch angle displayed by the protractor and plumb line.
Check that no one is in the line of fire.
Fire the launcher by pulling up on the trigger.
Watch the projectile as it travels and bounces so that you can retrieve it.
Place the catcher box so that the projectile lands into it on successive launches.

## Care of the launcher

Do not fire the launcher without a load. Such a "dry fire" may damage the launcher. If you wish to release a cocked spring without firing, insert the ramrod into the barrel against the cradle and,
while holding the ramrod firmly in place, release the trigger. Slowly ease the ramrod out of the barrel.

Do not over tighten the plastic wing nuts. They can easily strip and split.

## Measurements

## Projectile speed

Vertical launch
A projectile fired vertically should reach a maximum height depending only on its launch speed and the acceleration due to gravity. Measuring the apex of the trajectory gives you the information you need to calculate the launch speed.

## Horizontal launch

A projectile fired horizontally should hit the ground in a time depending only on its launch height (above the ground) and the acceleration due to gravity. Measuring the range of the projectile (the horizontal distance from launch to landing) and the launch height gives you the information you need to calculate the launch speed.

## General questions

How consistent is the muzzle speed from launch to launch?
How can you measure, quantify, and report the variability in muzzle speed?

## Range

What different launch angles should you use to create a complete plot of range vs. launch angle? How many different angles should you use, and how many shots at each angle?
How can you best estimate the launch angle that gives the greatest range?

## Processing, Analysis, and Interpretation

If you took multiple measurements of a particular outcome, how can you characterize its most likely value (the "estimate") and its variability? How can you find the parameters to use in the equations of motion that make up your model of the system? What parameters did you use, and why? How would errors in the parameters affect your model's predictions?
How can you "invert" your equations of motion to pinpoint the launch angle leading to the farthest landing distance? How reliable is the value you obtain this way?

## Lab Report

You need to write a real lab report, because you decide what measurements to take and how to analyze them. The report must contain the following sections.

## Abstract

Briefly describe the apparatus, measurements, and what you did with the measurements.

## Purpose

What is the educational value of this activity? What skills are you developing?

## Theory

This lab includes trajectories, mathematical models, max-min problems, and experimental variability, so there is no shortage of things to consider. What equations of motion apply to this situation? Where should a projectile with a given launch position and velocity land? How can you experimentally determine the projectile's muzzle speed? How can theory determine the launch angle giving the greatest range?

## Experimental

Describe the apparatus you used and the procedures you followed to find the muzzle speed of the projectile, and to find the effect of launch angle on range. Explain steps you took to minimize experimental and measurement error.

## Observations and Data

Your primary data should be recorded in your lab notebook. In your report, present the data in graphical form. (Graph your results, using a clear and informative format.) Discuss-and, if possible, illustrate-the variability in your data.

## Analysis and Discussion

Identify known or suspected errors in your measurements. Explain how these errors would affect your measured values.

Discuss your level of confidence in your estimate of $v_{0}$, the projectile's muzzle speed.
Explain how you obtained the predicted landing distances at the different launch angles. Discuss how well the theoretical predictions match your observations. Suggest reasons for mis-matches.
Identify and explain your best estimate of the optimal launch angle and its resulting range.
Discuss how well the theoretical prediction matches your observations. Suggest reasons for mismatches.

## Conclusion

- Explain your best estimate for the launch angle giving the greatest range for your launch height.
- Identify and discuss sources of uncertainty in your conclusions.
- Discuss how and why the optimal angle matches, or does not match, the theoretical optimal angle for a projectile landing at the same height as its launch.

