# Worksheet 27: Orbital Interactions

## *Circular orbit*

In a circular orbit, the centripetal force $mv^2/r$ on the orbiting body is the force of gravity $GMm/r^2$. This gives an orbital speed of $v = \sqrt{GM/r}$ and period $T = 2\pi r/v = 2\pi\sqrt{r^3/GM}$.

## *Bound and unbound orbits*

If the total mechanical energy $K_{tr} + U_g$ of two bodies in their center-of-mass frame of reference is negative, they are gravitationally bound. If positive, they will escape.

The escape speed from a gravitational attractor of mass $M$ depends on its mass and proximity: $v = \sqrt{2GM/r}$.

## *Closed orbits*

In a closed orbit, both bodies orbit their center of mass. Mechanical energy and angular momentum (about the center of mass) are both conserved.

The orbit of satellites orbiting a much larger attractor follow ***Kepler's laws:***

1. The orbits are ellipses with one focus at the massive attractor
2. The orbit sweeps out equal areas in equal times
3. The orbital period $T$ is related to the semi-major axis $a$ of the orbit as $T^2 \propto a^3$

# Problems

1. The mass of Earth is $5.97 \times 10^{24}$ kg.
   a. What is the escape speed from Earth's surface, $6.38 \times 10^6$ m from its center?

   b. What is the escape speed from Earth at the Moon's orbital distance, $3.84 \times 10^8$ m from Earth's center?

2. Suppose two solar-mass ($1.99 \times 10^{30}$ kg) stars make up a double system with a constant separation between them of 1 AU ($1.50 \times 10^{11}$ m).

   a. What is the period of their orbit?

   b. What is their centripetal acceleration?

3. The planet Uranus has a radius of 25,362 km and a surface gravity of 8.87 N/kg at its poles. Its moon Miranda is in a circular orbit at a distance of 129,560 km from Uranus's center. Miranda has a mass of $6.6 \times 10^{19}$ kg and a radius of 235 km.

   a. Calculate the mass of Uranus from the given data.

   b. Calculate the magnitude of Miranda's orbital acceleration due to its orbital motion about Uranus.

   c. Calculate the acceleration due to Miranda's gravity at the surface of Miranda.

   d. Do the answers to parts b and c mean that an object released 1 m above Miranda's surface on the side toward Uranus will fall *up* relative to Miranda? Explain what is happening.