Worksheet 25: Buoyancy and Bernoulli

Objectives

- Determine the buoyant force on an immersed object.
- Relate pressure, speed, and depth at different locations in continuous fluid flow.

Summary

Buoyancy

$$F = \rho g V$$

where F = upward buoyancy force, ρ = density of the fluid, and V is the displaced volume of fluid.

Fluid flow

Steady flow continuity condition: $dm_1/dt = dm_2/dt$

$dm = \rho dV$, so $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

Volume flow $dV/dt = \int_A \vec{v} \cdot d\vec{A}$; if $\vec{v}$ is constant throughout a cross section, $dV/dt = vA$

Steady flow of an incompressible fluid: $\rho_1 = \rho_2$, so $v_1 A_1 = v_2 A_2$

Bernoulli equation: $p_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$

where the fluid flows without branching (or the entire fluid flows past each point), A is the cross-sectional area of the fluid channel, V is a volume of fluid, m is its mass, v is its speed, p is its pressure, and y is its height.

Problems

1. A contractor's trick to make sure distant parts of a structure are at the same height is to partly fill a hose with water. When the two ends of the hose are held up, the water level is at the same height at both. Does it matter if there is an air bubble at one end of the tube?

2. A wood block with an iron weight attached to one face floats on water. Does it float higher or lower in the water if the iron weight is attached to the top of the block, out of the water, or to the bottom of the block, under the water?

3. A plinker fires a bullet into an open water tank, creating a hole that is a distance *h* below the water level. How fast does water emerge from the hole?

4. A tank resting on the ground is filled with water to a height *H* above the ground.
   a. A hole is tapped into the side of the tank at a height *h* above the ground. Water streams from the hole horizontally and then falls to the ground. At what horizontal distance from the side of the tank does the water strike the ground?

   b. Does the horizontal distance depend on the gravitational field *g*? Does this result make sense?

   c. At what height *h* must the hole be for the water emerging from the hole to land at the greatest distance from the tank?

5. Water emerges from a downward-facing tap with a diameter of 2.0 cm at a flow rate of 4.0 L/min. As the water falls, it speeds up and the stream becomes thinner. What is the diameter of the stream after it has fallen a distance *L* from the tap?