# Worksheet 14: Center of Mass

## Objective

- Find the center of mass of an object or set of objects.

## Summary

### Definition

The center of mass of a set of *N* point masses each with mass $m_i$ and position $\vec{r_i}$ is at $\vec{r}_{\mathrm{CM}} = \sum_{i=1}^{N} \frac{m_i \vec{r_i}}{m_i}$. The position of the center of mass of an extended object of mass *M* is $\frac{1}{M} \iiint_V \vec{r} dm$.

## Problems

1. Express the formula for center of mass in terms of the components ($x_i$, $y_i$, $z_i$) of the position vectors $\vec{r_i} = x_i \hat{\imath} + y_i \hat{\jmath} + z_i \hat{k}$.

2. Derive the formula for the velocity of the center of mass of a set of point masses.

3. A **center of mass frame of reference** has coordinate axes whose origin is at the center of mass of the system.

   a. The *j*th particle in the system has mass $m_j$ and velocity $\vec{v}_j$. What is its velocity in the center of mass frame of reference?

   b. What is the momentum of the *j*th particle in the center of mass frame of reference?

   c. What is the total momentum of all *N* particles in the center of mass frame of reference?

4. A 45.0-kg woman stands up in a 60.0-kg canoe that is 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end. Neglect drag from the water on the canoe.

   a. How far does the canoe move during this process? Assume that the center of mass of (canoe + woman) does *not* move.

   b. Relative to solid ground, how far does the woman move during this process?