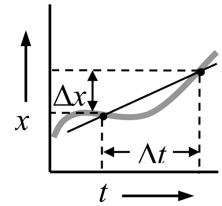
Worksheet 2: Straight-line constant acceleration kinematics

Summary

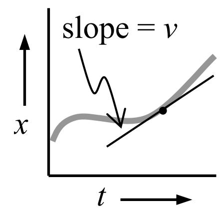


Average velocity is the ratio of how far an object moves to the time elapsed.

$$v_{avg} = \Delta x/\Delta t$$

On a plot of position vs. time, v_{avg} is the slope of the secant line connecting the starting and ending events.

Instantaneous velocity is the limit of average velocity as the time interval becomes infinitesimally brief. It is the velocity at a particular instant of time.



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

On a of position-time plot, v is the slope of the tangent line at that particular instant. Conversely, the area under a velocity-time plot is the change in position, $\int_0^t v dt = \Delta x$

Average acceleration is the rate of the change of an object's velocity.

$$A_{avg} = \Delta x/\Delta t$$

On a plot of velocity vs. time, a_{avg} is the slope of the secant line connecting the starting and ending events.

Instantaneous acceleration is the limit of average acceleration as the time interval becomes infinitesimally brief. It is the acceleration at a particular instant of time.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}; \qquad \int_0^t a dt = \Delta v$$

On a of velocity-time plot, a is the slope of the tangent line at that particular instant. Conversely, the area under an acceleration-time plot is the change in velocity.

Kinematic Formulas

When velocity is constant, dx/dt is the same for any time interval. Then $x = x_0 + vt$.

When acceleration is constant, $v = v_0 + at$; $x = x_0 + v_0 t + \frac{1}{2} a t^2$.

Algebraic substitution allows us to find relations not requiring t or not requiring a:

$$2a(x - x_0) = v^2 - v_0^2; \qquad x - x_0 = \frac{1}{2}(v_0 + v)t$$

1. A ball starts from rest and rolls down an incline at a constant acceleration. In 5.0 s, it rolls a distance of 50.0 m down the hill.

a. What is its acceleration?

b. If the same ball rolls down the same incline with the same acceleration, but begins with an initial speed of 2.0 m/s downhill, how far down the hill will it be in 5.0 s?

c. If the ball begins with an initial speed of 2.0 m/s *uphill*, where will it be in 5.0 s?

2. The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 30 m/s and you are stopped by an airbag that inflates from the dashboard, over what distance must you stop for you to survive the crash?

3. An automobile accelerates constantly from rest, traveling 400 m in 20.0 s.

a. What is its average velocity over this interval?

b. What is its final velocity?

c. The car's engine is adjusted, and the car is again accelerated constantly from rest through a distance of 400 m. This time, its final velocity is 50 m/s. What is the time elapsed?

2. A car 3.5 m in length traveling at 20 m/s approaches an intersection. The width of the intersection is 20 m. The light turns yellow when the front of the car is 50 m from the beginning of the intersection. The light will be yellow for 3.0 s.

a. If the driver steps on the brake, the car will slow at $-3.8\ \text{m/s}^2$. Will the car stop before the intersection?

b. If the driver steps on the gas, the car will accelerate at $2.3\ \text{m/s}^2$. Will the car clear the intersection before the light turns red?

13. A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2.

a. Sketch an *x*-*t* graph for both the student and the bus.

b. For how much time and for what distance must the student run before she overtakes the bus?

c. When she reaches the bus, how fast is the bus traveling?

d. The equations you used to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motion. Explain the significance of this second solution.

e. What is the minimum speed the student must have to *just* catch the bus?

f. If the student runs to just catch the bus, how far does she run to catch it?