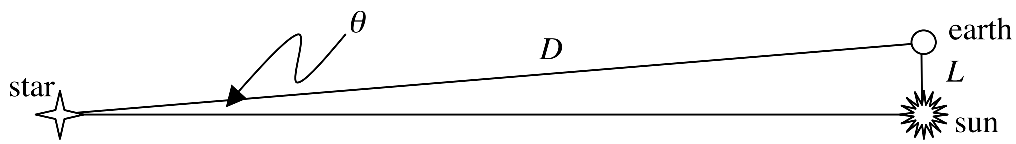
Measuring the Heavens: Parallax

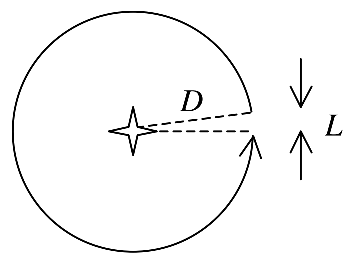
We define a right triangle by the position of the Sun, a distant star, and the earth at its position farthest from the sun-star line. (Not drawn to scale.)



The distance L from the sun to earth is the radius of the earth's orbit. The parallax angle θ is measured by sighting the star at different times of the year. We want to find the distance D to the star.

A complete circle is 360°. The angle θ is a fraction of that. Likewise, the sun-earth distance L is a fraction of a complete circle around the distant star. If θ is small (if L is a small fraction of the circumference of the circle), then the ratio of θ to 360° is exactly the same as the ratio of L to the circumference of the circle.

$$\frac{\theta}{360^\circ} = \frac{L}{\text{circumference}}$$



1. What is the formula for the circumference of a circle with radius D? ______________
2. Substitute the formula into the ratio to complete the equation below.

$$\frac{\theta}{360^\circ} = \frac{L}{\qquad\qquad}$$

3. Solve this expression algebraically for D. (Obtain an equation in the form $D = \ldots$".)

4. How does the height of side L compare to the length of side D if θ is a very small angle?

5. If a second triangle has the same height L but a greater width D, how does its parallax angle θ compare to the parallax angle θ of this triangle?

6. If a second triangle has the same parallax angle θ but a greater height L, how does the width D of the second triangle compare to D of this triangle?

7. The average sun-earth distance L is known as the **astronomical unit**, abbreviated AU. Rewrite the formula for D from question #3 with 1 AU in the place of L.

   $D =$

Parallax angles are very small, much smaller than 1°. Thus, astronomers express angles in terms of **arc minutes** (abbreviated ') and **arc seconds** (abbreviated "), where

- 60 arc minutes = 1 degree (60' = 1°), and
- 60 arc seconds = 1 arc minute (60" = 1').

8. Convert the 360° to arc seconds and crunch all the numbers you have (everything but D, θ, and AU) to simplify the formula from question #7. Using this formula, you can enter a star's parallax θ in arc seconds and find its distance D in AU.

   $D =$

9. The distance D to a star with a parallax angle $\theta = 1"$ is known as 1 **parsec** ("parallax second"), abbreviated pc. What is that distance in astronomical units?

   1 pc = ______________________ AU.

10. Stars are so far away that the AU is too short to conveniently describe their distances. Convert the formula from question #8 from AU to pc so that you can enter a star's parallax θ in arc seconds and find its distance D in parsecs. (It should be a *very* simple formula.)

    $D =$

11. If a star has a parallax angle of 0.2", what is its distance in parsecs? _________________ pc

12. 1 AU = 1.495979×10^8 kilometers. How far is a parsec in kilometers?

    1 pc = ______________________ km.

For problem #13, you will need the formula distance = (speed)·(time).

13. The speed of light is 2.997925×10^5 km/s. How many km does light travel in:

    a. 1 min = 60 s? _____________ km

    b. 1 h = 60 min? _____________ km

    c. 1 day = 24 h? ______________________ km

    d. 1 year = 365.26 days? ______________ km

The last distance is a **light-year**, abbreviated ly.

14. How many light-years are in 1 parsec? 1 pc = ______________ ly.